Fine grinding, a refresher

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ABSTRACT

Many power-based grinding models exist, and most operators are familiar with Fred Bond's "third theory". Bond's model is most commonly used to describe primary and secondary grinding to product sizes above, for example, 100 µm. Operators sometimes use Bond's equation to describe grinding in situations where it is not appropriate, such as fine grinding below 50 µm. Using an alternative model would be a better choice in this situation.

Bond's equation is one in a large family of models. Related equations better suited to fine grinding include the “signature plot”, Von Rittinger's model and Charles' equation. These models have a similar form to Bond's and can be fit to industrial regrind mills and laboratory tests using the simple regression tools in the charts of computer spreadsheets. Operators will find that fine grinding calculations are both easier and more accurate when using the alternative equations to fit their regrind milling surveys or when performing laboratory regrind mill scale-up for plant design.

None of these models are new: Charles' equation was published in 1957 and Von Rittinger's model was proposed in 1867. A quick refresh how to apply these equations can turn these oddities from the undergraduate curriculum into useful tools for plant optimization. They can be fit to any type of mill including stirred and tumbling ball mills.

KEYWORDS

Fine grinding, comminution, regrind, grinding models

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INTRODUCTION

Building a case for fine grind modelling requires reviewing some theory before building a framework for doing fine grinding calculations. The actual calculation procedure recommended is easy and can be applied by plant metallurgists. Don't be frightened by the theory; the end result is reasonably simple.

The observation that mineral breakage is related to the absorption of energy is usually attributed to Von Rittinger (1867). Lynch and Rowland (2005) describe that the ability to empirical test and calibrate energy models of breakage didn't exist until the widespread adoption of electric motors in the mining industry during the 1930's and 1949's. A series of technical papers published by the Allis Chalmers company during the 1940's demonstrate that they were attempting to do such a calibration using Von Rittinger's model which is usually stated as: "the energy consumed in the size reduction is proportional to the area of new surface produced."

The calibration effort ultimately failed, and in 1952 an employee of Allis Chalmers named Fred Bond proposed a different model which came to bear his name (Lynch & Rowland, 2005). Bond's work index model was empirically fit to the wide variety of data collected over the previous decades and was particularly focused on the ball and rod mills that were in common use at the time (Bond, 1952).

Bond's model wasn't immediately accepted by the mineral dressing industry, and its applicability was debated for about a decade after publication. Alternative models appeared, such as the model by Charles (1957) where an ore-specific coefficient and exponent on the size term would be measured. The debate largely settled when Hukki (1962) published a reconciliation of the major competing models of the time and suggested that all models were valid, but each within a certain size range.

A crucial observation of Hukki is that Bond's equation is generally only valid in the range of primary and secondary grinding (product sizes between 1 mm – 100 µm), and that different models apply below this size range. This paper will discuss how to use those alternative models in an industrial setting.

Hukki's Conjecture

Hukki performed some simple crushing and grinding tests at his laboratory in Helsinki where he mapped the consumption of specific energy from a "large" size, roughly representing a primary crusher discharge, down to as fine a size as was easily achieved in his laboratory. The result is shown in Figure 1 where the coarse sizes absorb relatively little energy to break to smaller sizes, whereas the finest size absorb a great deal of energy to break. Some component of this observed energy is likely the inefficiency of the apparatus used, but it is reasonable to assume that the largest component of the increasing energy requirements at fine sizes is due to the comminution energy being absorbed by the ore.

Hukki proposed that all power-based comminution models descend from a single relationship, shown in Equation (1). The variable exponent $-f(x)$ is required because of the varying slope observed in Figure 1.

$$\frac{dE}{dx} = -K x^{-f(x)} \qquad (1)$$

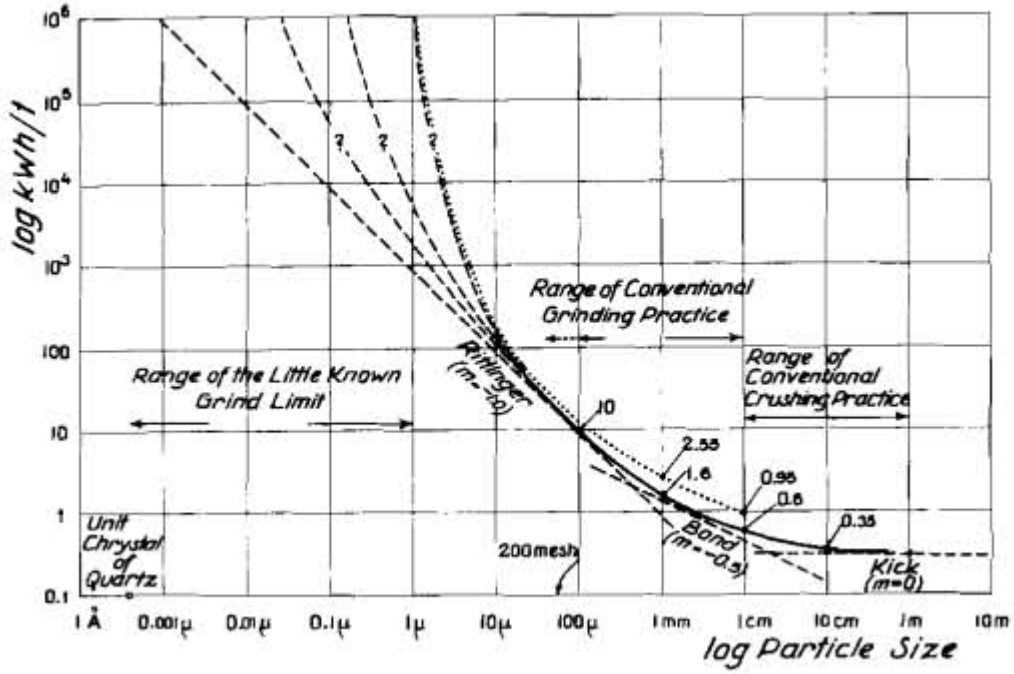

Figure 1: Hukki's Conjecture (Hukki, 1962)

For a sufficiently narrow range of sizes, the exponent can be assumed to be constant. One solution of this differential equation is given in Equation (2):

$$E = C\left(P_{80}^{-\alpha} - F_{80}^{-\alpha}\right) \qquad (2)$$

In this form, it is obvious that Bond's equation results if $C = (10\ Wi)$ and $\alpha = ½$. If the exponent α is measured experimentally, then Equation (2) becomes Charles' equation.

Hukki's Conjecture includes the observation that the slope, hence the exponent $-f(x)$, varies with particle size. Roughly measuring the slopes in Figure 1 and plotting at the mid-points of the base-10 logarithms gives the variation of the exponent shown in Figure 2.

In Hukki's example, Bond's equation with exponent of -½ fits grinding to the 1 mm size range, roughly the product size expected from a rod mill. The exponent in ball mill size range, around 100 μm, is close to -1, the exponent predicted by Von Rittinger's model. This is only one result (Hukki acknowledges it is "hypothetical") and other ore types tested by other authors can give different curves, such as the three results from Levin (1989) for a CuNi matte (H1004), a uranium ore (H679) and a ferrochrome sample (J769).

The exponents from Levin's work vary for each material, and along with the coefficient, can be considered as a measurable property of a material. This suggests that any model of fine grinding requires two measured parameters, a coefficient and an exponent.

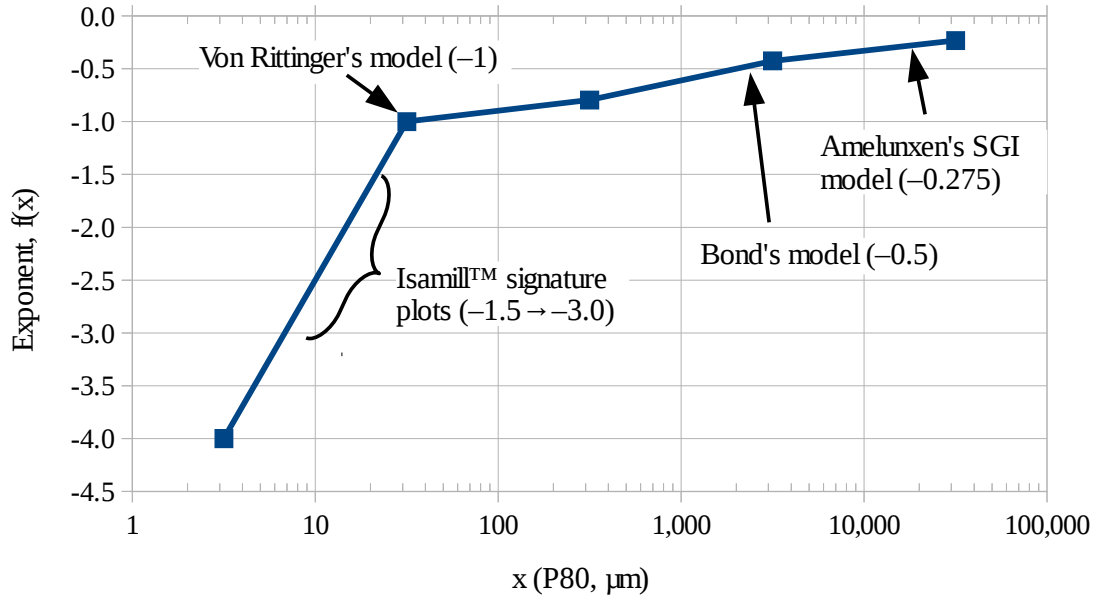

Figure 2: Variation of Hukki's exponent *-f(x)* with particle size

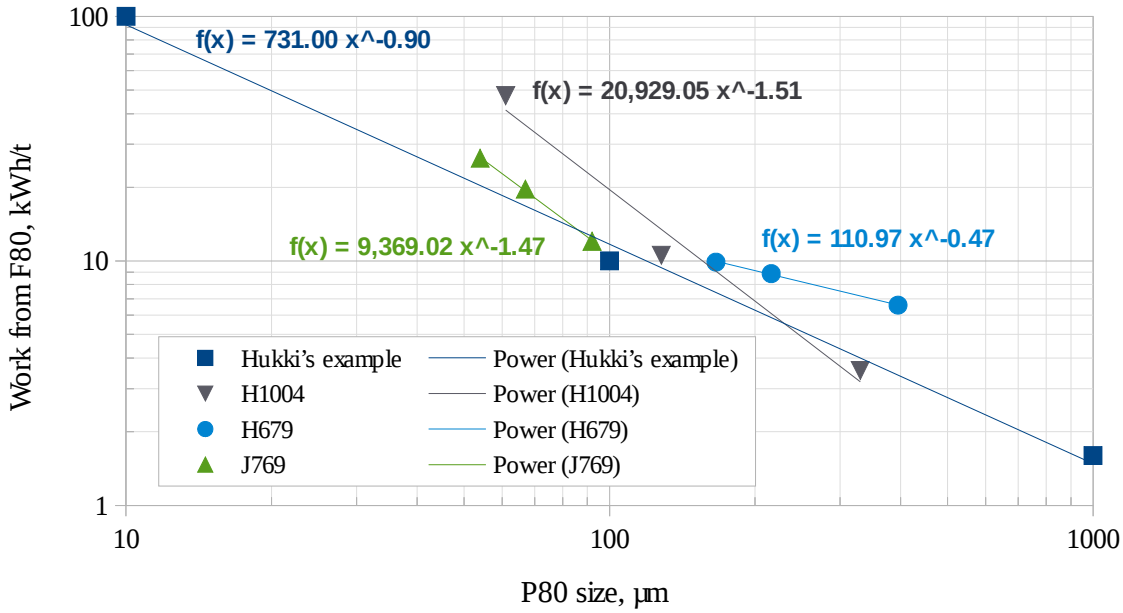

Figure 3: Comparing Hukki's example to data by Levin (1989)

METHODOLOGY

A fine-grinding model can be constructed using a chart in the form of specific energy versus P_{80} size, as in Figure 3, and the regression functions your preferred spreadsheet. At least two points are needed to construct such a chart, and if two surveys are not available then a usable model is still possible by assuming a suitable exponent.

The equation used in fine grinding is shown as Equation (3). The P_{80} term overwhelms the F_{80} term in Equation (2) at fine sizes, so the feed size is negligible and can be dropped. Moreover, in a regrinding application, the concentrate feed size is usually fixed and does not vary with different regrind mill circuit equipment or configuration.

$$E = C\left(P_{80}^{-\alpha}\right) \qquad (3)$$

Laboratory test results

The easiest way to construct a fine grinding model is using laboratory test results. The "signature plot" result of an Isamill™ laboratory test is already in the form of Equation (3) and can be used directly for modelling. The Levin test, performed using a dry Bond ball mill work index apparatus, also gives a result in the form of Equation (3). Both these tests provide a coefficient C and an exponent α measured for a particular sample.

Two other commonly used tests can be re-formulated to give estimates of fine grinding models in the form of Equation (3). The Jar Mill test favoured by Metso gives a table of results of E and measured size x for a progressive batch grinding test; these results can be fitted as demonstrated in Figure 4.

x, µm	E, kWh/t
110	0
53	3.38
38	6.75
33	8.44

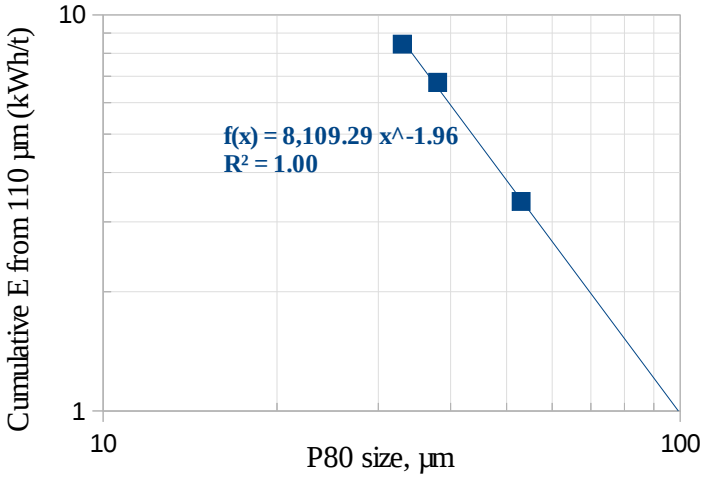

Figure 4: Jar mill test result, Merriam et al, 2015

A series of Bond ball mill work index tests run at different closing mesh sizes can be fitted in almost the same way. Work out the specific energy from the test feed size to each test's product size (solve

Bond's equation), then plot the resulting *E* versus product size *x* as demonstrated in Figure 5. Reminder that this will not be valid if you test "ore" but are modelling the regrind of "concentrate"; you need to test the concentrate.

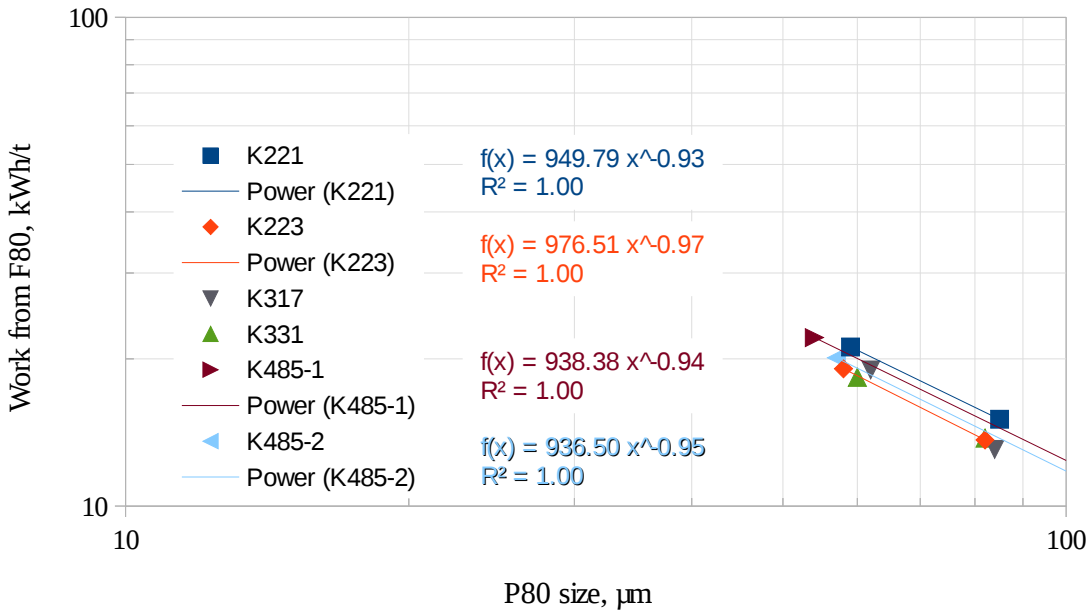

Figure 5: Fitting data from Aureus Mining (2012) Bond ball mill tests on gold ore

Existing Operations

Fine grinding is often conducted to regrind a rougher flotation concentrate prior to cleaning. As the concentrate is not the same composition as the ore ground in earlier comminution stages, it must be considered in isolation from the primary and secondary grinding circuits.

The two-parameter procedure used in the laboratory is usually impractical in an operating plant because it is not possible to vary the grinding energy in a regrind circuit (where you could then measure the change in P_{80}). If you can run a couple of tests at varying energy input and P_{80} sizes, then you should certainly do that. If you cannot, then you can assume an exponent (see Table 1 for suggested exponents) and use a single regrind survey (measuring specific energy at the mill pinion *E*, kWh/t and product P_{80} size *x*, µm) to solve for the coefficient.

Table 1: Suggested exponents for fine grinding applications

Material	Exponent	Equation	Size range
Gold ore (hydrothermal, greenstone, silicate hosted)	-0.9	$E = C\,x^{-0.9}$	500 → 40 µm
Lead-zinc ore (massive sulphide)	-1.0	$E = C\,x^{-1.0}$	65 → 45 µm
	-1.4	$E = C\,x^{-1.4}$	45 → 5 µm
Porphyry ore (silica, feldspars, minor sulphides)	-0.5	$E = C\,x^{-0.5}$	235 → 78 µm
Copper rougher concentrate (chalcopyrite and pyrite)	-1.5	$E = C\,x^{-1.5}$	110→33 µm
Pyrite concentrate	-2.0	$E = C\,x^{-2.0}$	40 → 8 µm
Base metal matte (copper, nickel)	-1.5	$E = C\,x^{-1.5}$	300 → 60 µm
Iron ore (hematite, magnetite)	-0.7	$E = C\,x^{-0.7}$	160→75 µm
	-1.8	$E = C\,x^{-1.8}$	75 → 15 µm
Zinc concentrate (Gao et al, 2007)	-1.2	$E = C\,x^{-1.2}$	20 → 5 µm

Discussion, different classes of stirred mills

The measured specific energy consumption in grinding is a combination of the energy required to break apart the rock and the energy consumed (and wasted) by the mill. For the purposes of this discussion we will assume that all “efficient” machines will give a similar model coefficient and exponent. The reality is that certain machines will be more efficient in a particular size range; this is due to a variety of factors including smaller media being better suited to fine grinding, differences in classification efficiency and circulating loads (internal to the machine and external), and so on. Figure 6 shows an example of a comparison of fine grinding efficiency of two classes of equipment written by the vendor of the “blue” equipment that demonstrates superiority over the “red” equipment. The “red” vendor presumably has similar diagrams showing the opposite conclusion.

If you build a model for your particular type of equipment on your material at the size range that you expect to be working with, then that model should be valid for modelling. If you change from a very inefficient type of equipment (example, a tube mill) to one that is more efficient (a stirred mill), then you should update your model.

The Author suggests that all modern stirred mills can be considered equivalent for conceptual design in the typical regrind size ranges (40 to 75 µm). Equipment vendors may have different opinions.

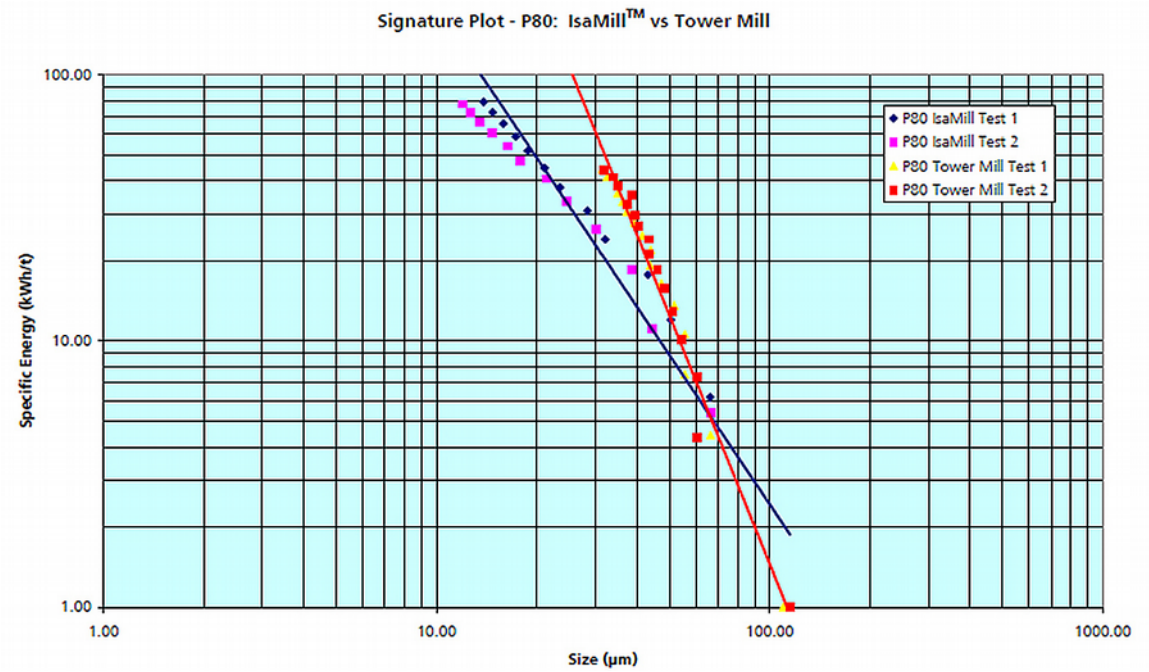

Figure 6 – Ernest Henry Mining small scale test work (Burford & Niva, 2008)

CONCLUSIONS

- The widely-used Bond equation was based on curve-fitting a large set of data. The size term $(1/\sqrt{x})$, which corresponds to an exponent of –0.5, is just an average of a large data set.

- Hukki's Conjecture is that the exponent on the size term varies with size and material. Bond's equation is a particular case where the material is "typical ore" and the size range is around 1 mm.

- Fine grinding exponents should be material-dependent, and often particle size dependent. Rarely do materials demonstrate Bond's exponent of –0.5 below 100 µm.

- Predictions of performance of fine grinding equipment should not be based on Bond's equation; use an alternative equation (Equation 3) with a measured, material-specific coefficient and exponent.

WORKED EXAMPLES

Laboratory test result, conceptual design

A conceptual regrind stirred mill design based on the laboratory jar mill test result shown in Figure 4 (which already includes a 0.65 factor for stirred milling) would have the model equation: $E = 8109\ x^{-1.96}$ to regrind a rougher concentrate to a specified P_{80} value of x. This model is valid for F_{80} of 110 µm and P_{80} in the range of 53 µm to 33 µm. Given the design criteria of:

- a desired product size of 45 µm;
- a desired regrind circuit throughput of 10 tonnes per hour; and
- a desired design allowance of 15%.

The required stirred mill motor output power would be $8109\ (45)^{-1.96} \times 10 \times 1.15 = 54$ kW

Operating plant, adding capacity

An operating plant has two small regrind ball mills in parallel treating a copper porphyry concentrate and wants to predict the grind that can be achieved by adding a third identical regrind ball mill in parallel with the existing two. A mill survey was conducted where 43 dry t/h of combined regrind mill feed (F_{80} = 180 µm) consumed 483 kW + 474 kW (measured at the motor input) and resulted in a 63 µm cyclone overflow P_{80} size.

- First correct the motor power so it represents the power at the mill shell. The motor name-plate says the motor efficiency is 0.958 and allow 0.985 efficiency for the pinion & gear. Motor power at the mill shell = (483 + 474) × 0.958 × 0.985 = 903 kW.
- Specific energy consumption, E = 903 kW ÷ 43 t/h = 21 kWh/t
- Select the exponent (-1.5) from Table 1
- Construct equation (3): $E = C\ (x)^{-\alpha}$; $21 = C\ (63)^{-1.5}$
- Solve for C: $C = 21 \times (63)^{1.5} = 10{,}501$ (unitless)

Now add the third regrind mill. Because the feed rate is the same and there is more power available, E becomes (21 × 3/2) = 31.5 kWh/t. Now solve for x using the value C determined in the survey.

- $E = C\ (x)^{-\alpha}$; $31.5 = 10501\ (x)^{-1.5}$
- $x = (31.5 \div 10501)^{-(1/1.5)} = 48$ µm

If this calculation were performed using the Bond operating work index (OWi = 40.8 kWh/t), the predicted size would be 43 µm.

Operating plant, higher throughput

Use the same example plant as above with the same survey data. What would be the effect on the grind size of the two existing regrind mills if the throughput were to increase to 50 t/h?

- The same regrind mill power is available, 903 kW at the shell. The higher throughput means the specific energy consumption will decrease: E = 903 kW ÷ 50 t/h = 18.1 kWh/t

- $E = C\,(x)^{-\alpha}$; $18.1 = 10501\,(x)^{-1.5}$; x = 70 µm.

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