INTRODUCTION TO ROTORDYNAMICS OF PUMPS WITHOUT FLUID FORCES

by

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INTRODUCTION

This paper is for the user of rotating equipment. Reliability is the key to the bottom line and rotordynamics is often the significant factor in determining reliability. Rotordynamics is a very interesting and complicated subject. The importance of this subject has increased over the last few decades, particularly in smaller equipment such as pumps. As machine speeds have increased and higher flows and efficiencies have become necessary, the side effect has been to introduce new rotordynamics problems. These include critical speeds, unbalance response and rotor stability. An example of a recently seen problem involved the application of variable frequency drives to vertical pumps which induced structural resonances in the system at operating speeds. The object of this paper is to discuss various aspects of rotordynamics that are general and then apply these to some examples. The mathematics will be kept to a minimum and as many helpful "rules of thumb" will be included as this subject allows.

Rotordynamics can be a very controversial subject from the nomenclature used to the question of the degree of accuracy needed to model a rotordynamic system. There have been many simplifications and assumptions made in this paper and the author offers some opinions that some people will disagree with, but the approach here is quite conservative and the guidelines will not get anyone into trouble. The approaches and guidance offered here are based on experience of many people and many years of analyzing and testing machinery. The track record of those using these techniques is very good when comparing analytical and actual test data.

The paper is broken down into two major sections. The *first section* covers rotordynamics of machines with the majority of the mass between bearings such as a compressor or a multistage pump. All the effects of the various geometrical factors will be discussed for these types of machines. The *second section* covers overhung mass machines with a particular eye toward the rotordynamics of overhung pumps.

A nomenclature page is included and, whenever possible, the symbols used for various factors are identified in the body of the paper.

Rules of thumb are fine, but the reader should come to the conclusion that the only way to do a meaningful rotordynamics analysis on a machine train will be to procure the necessary computer programs to do the analyses. The programs used by the author were all developed by the University of Virginia. There are many other sources for computer programs and most of these will run on desktop computers that free the engineer from the chains of a mainframe computer and make the analysis easier and faster.

In the future, as computers become friendlier and more powerful, the design engineer will be able to analyze complete *systems,* including such things as rotor-structure interaction, effects of fluid forces and internal wear, process upsets and other effects. Since *basic principals* will not change, this paper can serve as a guide for the necessary steps in analyzing a piece of rotating machinery.

GENERAL ROTORDYNAMICS

A compressor rotor, pump impeller, steel structure, and a tuning fork all have something in common: they all have resonances. When a tuning fork is struck, it emits a tone (actually many tones, but one *principal* tone). Strong winds may "ring" a buildings's natural frequencies or cause large motions on a bridge. A rotor may have its natural frequencies excited by many sources: rotating unbalance, vane-pass excitation in a pump, rubs, or process changes such as surge or flow instabilities caused by operation too far away from BEP. The *first objective* of rotordynamics is to identify the resonance frequencies present in a *system* and design the system around them. A typical machinery train may consist of a driver, a gear and a driven such as a compressor. A motor or turbine driven pump has baseplate and other structural resonances. It is not atypical to have 10 or more system resonances to design around. So how does one begin? Does one need a complicated computer analysis to identify these resonances and do a complete system analysis? The answer is, yes, you probably do. However, there are many things that can be looked at quickly and easily with an eye toward general trends and design practices. In some cases, this may suffice, particularly if the vendor is capable of good rotordynamics analysis. The user will find that compressor vendors, for example, are very open about their rotordynamics capabilities, but most builders of smaller equipment (particularly pumps) either have no rotordynamics program, or it is invisible to the users. Small equipment has been severely overlooked in the past (unless a major problem arose), simply because it was too time consuming, too expensive, or simply deemed unnecessary. Today, analysis techniques are commonly available to the design engineer at his desk and there is *no excuse* for not doing a routine analysis on new designs and even reviewing older designs that the *users* have fixed in the field. The best defense a user can have is to develop his own rotordynamics capabilities and *cross-check* important machinery against the design of the vendor before purchase.

A typical rotor, supported in bearings of some sort, is analogous to the familiar spring-mass-damper system shown in Figure 1. The governing equation for the motion of such a system is:

$$M\ddot{x} + K\dot{x} = \text{[the forcing function]}$$

What this means is that the forcing function (which in a rotating element usually means the unbalance forces or in a pump it could be vane-pass pulsations) is opposed by the system *inertia,* the system *damping* and the system *stiffness*. When the frequency of a forcing function coincides with a natural frequency of the rotor, we encounter what is commonly called a *Critical Speed*. If we ignore the damping term for the moment and set the forcing function to zero, we find that the solution of the equation gives the first natural frequency:

$$\text{Natural Frequency} - \omega_n = \sqrt{K/M}$$

If damping is included the solution is:

$$\omega_n = \sqrt{C/2M + (C/2M)^2 - (K/M)}$$

These solutions are not directly applicable to rotor systems, as we shall see. In a simple example shown in Figure 2, consider a single mass rotor with a rigid shaft supported on identical bearings:

$$\begin{aligned}\omega_n = \sqrt{K/M} &= \sqrt{(1{,}000{,}000\ \text{lbf/in})/1000\ \text{lbm} \times (386\ \text{in-lbm/lbf-in}^2)}\\ &= 621.13\ \text{rad/sec} \times 1\ \text{Cycle/[}2 \times \text{pi rad]} \times 60\ \text{sec/min}\\ &= 5{,}933\ \text{rpm}\end{aligned}$$

However, a rigid shaft is not normally a reasonable assumption, and many shafts have more than one mass, either between bearings or overhung. Thus, we must first find the shaft stiffness from beam theory.

For a circular shaft:

$$K_s = 48EI/L^3$$

where $I = \text{pi} \times D^4/64$

So, for example, take a 5 inch diameter shaft with a span of 80 inches:

$$\begin{aligned}K_s &= [48 \times (30 \times 10^6) \times (\text{pi} \times 5^4/64)]/80^3\\ &= 86{,}286\ \text{lb/in}\end{aligned}$$

Now to include this in the previous example, we must calculate the *system* stiffness. Springs in series add like resistors in parallel, that is, inversely. So:

$$\begin{aligned}1/K_{system} &= 1/(2 \times K_{bearing}) + 1/K_{shaft}\\ &= 1/1{,}000{,}000 + 1/86{,}286\\ K_{system} &= 79{,}432\ \text{lb/in}\end{aligned}$$

Notice that the stiffness is *always* lower than any of the component stiffnesses. Thus, our natural frequency calculation is now (assuming that the total system mass is the same, 1000 lbm):

$$\begin{aligned}\omega_{n\ system} &= (79{,}432 \times 386)/1000\ \text{rad/sec}\\ &= 175.1\ \text{rad/sec} = 1{,}672\ \text{rpm}\end{aligned}$$

This is a 255 percent *decrease* from the rigid case showing that shaft stiffness is the dominant factor here. This principal was originally developed in 1894 by Dunkerly, who stated that the principle of superposition applied to critical speeds in an inverse manner:

$$1/\omega_{n\ system} = 1/\omega_{n\ shaft} + 1/\omega_{n\ mass1} + 1/\omega_{n\ mass2} + \ldots$$

Additional information about the mathematical approach to rotordynamics is availble in abundance.

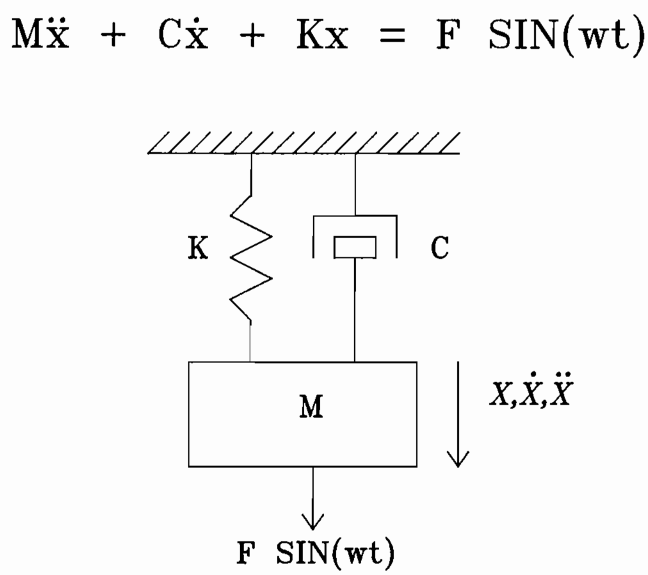

Figure 1. Simple System.

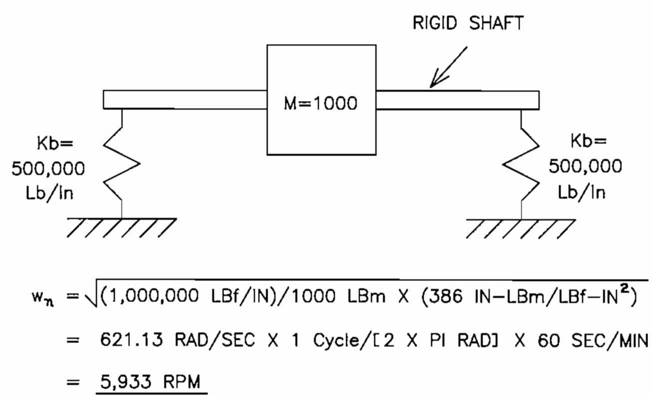

Figure 2. Rigid Shaft System.

To get a feel for a given rotor-bearing system, some general rules of thumb can be applied. The first of these is the correct of *modal mass.* Modal mass is the effective mass "seen" by a particular mode at resonance. For a between bearing system at its first critical speed, the modal mass is equivalent to the mass that would yield an equivalent system if all the mass was lumped in the center of the span. A plot of modal mass ratio (modal mass/total mass) as a function of the stiffness ratio between bearing and shaft stiffness ($2 \times K_b/K_s$) is shown in Figure 3. The stiffness ratio is very important and greatly affects the modal mass ratio among other things. In smaller machinery, such as single stage overhung process pumps, the effective stiffness "seen" by the rotor is greatly influenced (reduced) by the foundation. The mass of the structure is also impotant in influencing the response of the system, when the rotor mass is on the same order as the case mass. As the support stiffnesses become much less than the shaft stiffness, the rotor begins to behave as a free body and the modal mass becomes equal to the total mass (ratio on 1). As the support stiffnesses became much larger than the shaft stiffness, the modal mass ratio asymptotically approaches 0.5. For most actual cases involving large compressors and turbines that this author has encountered, modal mass ratios between 0.55 and 0.65 have been most common. When in doubt, use 0.6. (No rule-of-thumb for smaller equipment is applicable here due to structure interaction). For an example of a large machine, see Figure 4. This is an actual propylene refrigeration machine installed in an ethylene plant. Suppose we need a quick estimate of this machine's first critical speed so we don't accidentally run too close to it during a startup. We begin by calculating the shaft stiffness:

$$K_s = 48EI/L^3 = [48 \times (30 \times 10^6) \times (\text{pi} \times 14.25^4)/64]/147^3$$

$$K_s = 917{,}600 \text{ lb/in}$$

The bearings for this rotor are highly preloaded tilting shoe bearings and were calculated to have a stiffness of 3,500,000 lb/in each. So to get the system stiffness, we add the stiffness inversely:

$$1/K_{system} = 1/[2 \times K_{bearing}] + 1/K_{shaft}$$
$$= 1/7{,}000{,}000 + 1/917{,}600$$
$$K_{system} = 811{,}200 \text{ lb/in}$$

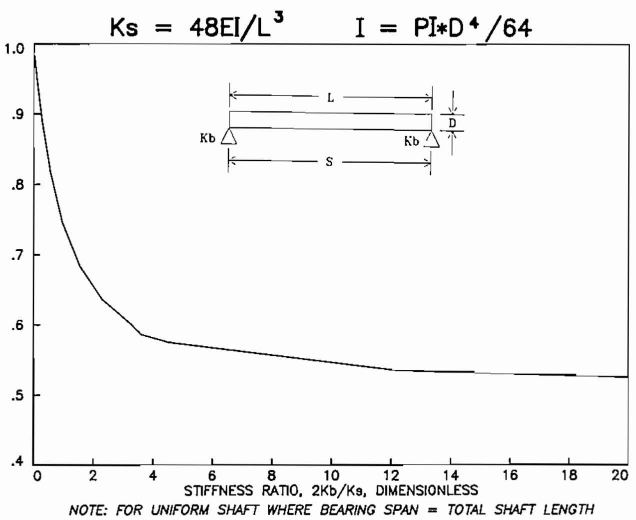

Figure 3. Effect of Shaft-Bearing Stiffness Ration on Modal Mass at First Critical—Plain Shaft.

If we were to use the *total* rotor mass to calculate the first critical speed:

$$\omega_n = \sqrt{K/M} = \sqrt{(811{,}200 \times 386)/11{,}959}$$
$$= 161.8 \text{ rad/sec} = 1{,}545 \text{ rpm}$$

But this is not even close to the actual critical speed. So, refer to Figure 3 and calculate the stiffness ratio 2Kb/Ks = 7.63. This translates to a modal mass ratio of 0.56. Thus, our first critical speed calculation becomes:

$$\omega_n = \sqrt{K/M} = \sqrt{(811{,}200 \times 386)/(0.56 \times 11{,}959)}$$
$$= 216.2 \text{ rad/sec} = 2{,}065 \text{ rpm}$$

As can be seen in Figure 5, the actual first critical speed of this machine was between 2000 and 2100 rpm. Thus, we can see that this method works fairly well for between bearing rotors with evenly distributed external masses (wheels).

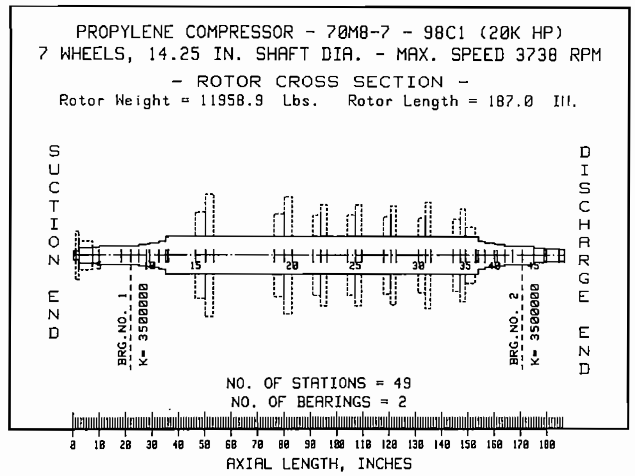

Figure 4. Actual Compressor Model.

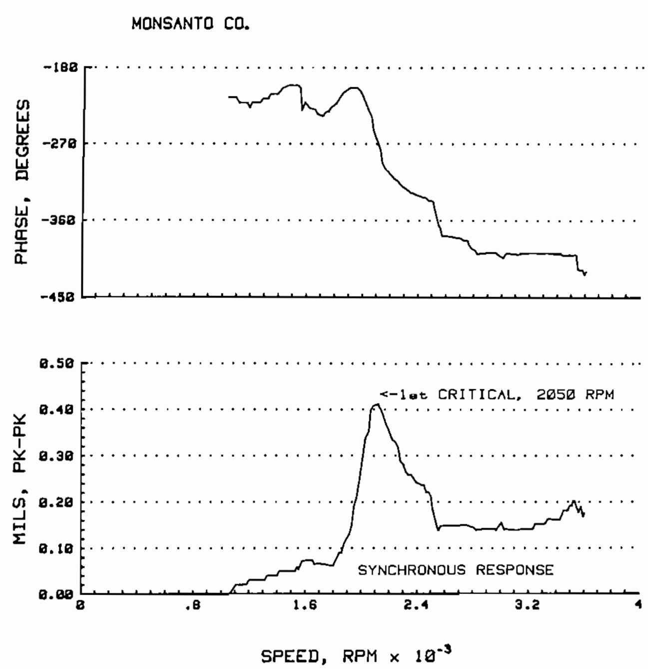

Figure 5. Actual Compressor Response.

The concept of *stiffness ratio,* $2K_b/K_s$, is useful in other ways. It can be used as an indicator of rotor sensitivity to unbalance forces and also a measure of inherent stability. In general, the higher this ratio, the more sensitive and the less stable a rotor becomes. Some of the reasons for this will become evident in the next section. A good rule of thumb is that a *stiffness ratio greater than 10* should start to cause concern. Indeed, real machines with stiffness ratios greater than 10 cause operations and maintenance people great concern all the time. Rotors like this are called "flinky" or "noodles," among other things. Typically, flexible rotor-bearing systems that are well designed (and well behaved) will have stiffness ratios ranging from 4 to 8. Stiffness ratios in pumps are greatly influenced by the bushings and wear rings, and the respective clearances and pressure differentials across them.

Another factor related to rotor sensitivity is commonly called the *"Q" factor.* This term, borrowed from the electronics field, refers to the sharpness of a system resonance. "Q" can be defined several ways; the two most common are:

$$Q = \frac{\text{Amplitude at Resonance}}{\text{Amplitude at a Speed Much above Resonance}}$$

or:

Q = Amplification Factor as defined by API
= Nc1/(N2-N1)

where:

Nc1 = the frequency at resonance

N1 = the frequency below resonance where the amplitude is 0.707 times the amplitude at resonance (half power point)

N2 = same as N1 except the half power point frequency above resonance

A good rule of thumb here is, as the API states, the "Q" or Amplification Factor *should not exceed 8,* and values below 5 are preferable.

Finally, the concept of logarithmic decrement is important. Often called the log dec for short, this factor is related to "Q" and is defined as the natural log of the ratio of two successive resonant amplitudes, as can be seen in Figure 6. When the log dec is positive, the system's vibrations die out with time and the system is stable. However, if the log dec is negative, the system's vibrations grow with time (when an excitation exists) and the system is unstable. The relationship of the log dec (when positive) to "Q" is this:

$$Q = \text{pi}/(\text{log dec})$$

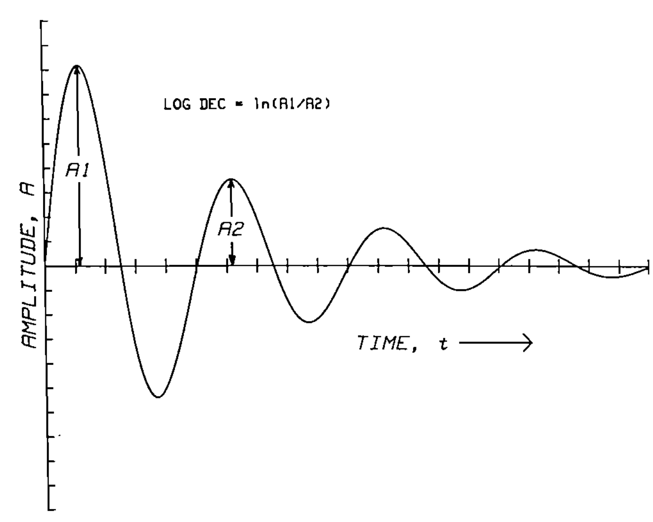

Figure 6. Logarithmic Decrement.

The log dec can be experimentally determined by momentarily exciting a running rotor at one of its natural frequencies, either by a forcing function or an impulse. By recording the resultant "ring-down," the log dec can be calculated. Compressor surge is an excellent exciter (and wrecker) of machinery, but if well controlled, this may be used to impulse the rotor and excite the first critical while running at a higher speed. This is shown in Figure 7, which is the time trace of an axial compressor's vibration signal during a surge test. A log dec of 0.35 was calculated from this trace as shown.

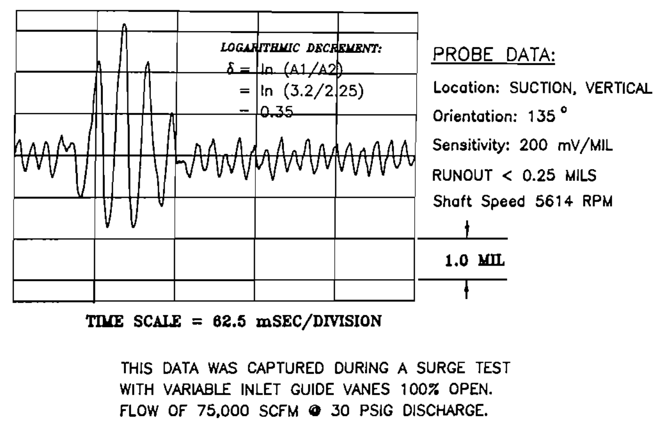

Figure 7. Resonant Excitation of Axial Air Compressor Induced by Surge.

THE ELEMENTS OF ROTORDYNAMICS

The approach of this section will be to examine how various elements such as the bearings, the shaft and the external masses on the shaft affect the system's critical speeds. Most of the machinery in our refineries, petrochemical plants, etc. falls into one of two categories: small "stiff" shaft machinery, like process pumps, or large "flexible" shaft machinery, like a compressor or multistage turbine. Unfortunately, the lines of demarcation are not well defined and some equipment, like a multistage pump, might be considered either "stiff-shafted" or "flexible," depending upon the internal close clearances and the degree of wear present. The difference is that "flexible shaft" machinery goes through one or more critical speeds on its way to operating speed. The vast majority of the "flexible shaft" machinery in operation, such as are found in typical process plants (compressors, turbines, etc.), goes through the first critical speed upon startup and runs between the first and second critical speeds. Most pumps are not designed to run above their first critical speed, but there are complications, such as excessive wear ring and bushing clearance, that can cause this to happen. The type of machinery we are most interested in analyzing *does* go through critical speeds. One may ask, why look past the first critical, if the second critical and above are seldom encountered? You must know where the second critical speed is so your speed boundaries are set and the second critical speed may excited by asymmetries (such as a flat spot) in the shaft at lower speeds. The third critical must be known because there may be other exciting forces in the system like vane-pass frequencies. *Generally,* the modes higher than the third will not be of consequence except in unique situations.

Each rotor, regardless of type, is supported on some type of bearings which are usually either hydrodynamic or anti-friction. In turn, the bearings are supported by a bearing housing and pedestal, a baseplate, a concrete foundation and finally, Earth. The effective stiffness of the rotor support "seen" by the rotor is the sum (remember the inverse additive property of springs in series) of all these "springs." This is one reason one cannot assume extremely high stiffness values for anti-friction bearings: the bearing housing and support feet become the "soft" member, particularly in the horizontal direction. Bearing design and application is a complicated subject and will not be covered in depth here. There are many types of journal bearings, each of which has particular strengths and weaknesses. For example, tilting pad bearings (which are often broadly applied as a panacea for all that ails turbomachinery), have the disadvantages of high horsepower consumption and less damping ability than a plain journal bearing of the same load carrying capacity. On the plus side, they are very stable. From a rotordynamics standpoint, the principal factors of bearing design are the stiffness and damping characteristics, their applied location (bearing span), and their stability. For now, let's individually concentrate on *bearing stiffness, damping* and *span* and their *interaction.*

The Effect of Support Stiffness —Between Bearing Systems

Continuing with the original example, a plain shaft (many rotors have well-distributed external masses and can be approximated by a plain shaft—mass distribution will be examined later) with a 5 inch diameter made of high quality steel, a bearing span and a total length of 80 inches. The shaft stiffness is K_{shaft} = 86,286 lb/in. What happens as bearing stiffness varies from 10,000 lb/in, which is very "soft," to a very rigid bearing of 10,000,000 lb/in? A *very important* plot which you should become familiar with—the undamped critical speed map, is shown in Figure 8. This plot shows how the first three critical speeds are affected by support stiffness. Note that the first two critical speed curves have similar shapes, increasing rapidly in value at low bearing stiffnesses until the stiffness reaches approximately 1,000,000 lb/in and the curves start to become asymptotic. No matter how stiff the bearings, the first critical speed for this rotor can never exceed 3,660 rpm and the second critical can never exceed 14,360 rpm. Notice that below about 1,000,000 lb/in support stiffness, variations in stiffness will yield larger changes in the first two criticals than above 1,000,000 lb/in. For this reason, it is often advantageous to design rotor-bearing systems to have characteristics such that the critical speeds fall on the sloped portion of the curve. Suppose after a machine is installed, the process needs to have the machine run faster or slower, or some mechanical reason, such as a piping resonance, dictates a speed change? If the machine is already on stiff supports, there may be no way to change speeds and still stay away from the critical speeds. However, if the bearings are not too stiff, an upgrade may push the criticals sufficiently high to allow a speed increase.

The third critical speed, often called the "free-free" mode or "bearingless" mode, is unaffected by stiffness changes at low support stiffnesses. However, as the bearings begin to "clamp down" at higher stiffnesses, more bending is introduced into the rotor and the third critical speed will rise. The third critical speed is rarely encountered in actual machinery, because it can destroy a rotor due to the complete stress reversal with each revolution. Its measurement can be a useful tool to check the accuracy of a theoretical critical speed program. Supporting a rotor on wires or other very soft support and ring-testing, either with an exciter or a hammer blow, will "ring" the rotor at its zero support stiffness (no bearings) third critical speed.

Now, having seen how support stiffness affects the frequencies at which critical speeds occur, let's look at how the rotor itself is affected. The *modeshapes* for the first critical speed of our example rotor for soft and hard support stiffnesses and shown in Figures 9a and 9b. A theoretical modeshape is essentially a prediction of the *degree* of bending occurring in a rotor at a critical speed and *where* that bending is occurring. The amplitude is nondimensional, since the unbalance will determine the actual amplitudes and this analysis does not include that. Simply think of the maximum amplitude as "100 percent" and the other points on the curve as a percentage of the maximum. With the softer supports, the first critical speed is almost cylindrical with only slight shaft bending. This is often called a *translational* mode, as is the second critical speed. With very stiff supports, the shaft undergoes much more bending to the point that *node points* occur in the bearings. Nodal points are points of zero motion at the critical. At first glance this may seem ideal: while going through the critical, the bearing "sees" no motion! Actually, this is the worst condition we could have. For one thing, vibration probes are often placed adjacent to the bearings and if the bearings are very stiff, you will not have a

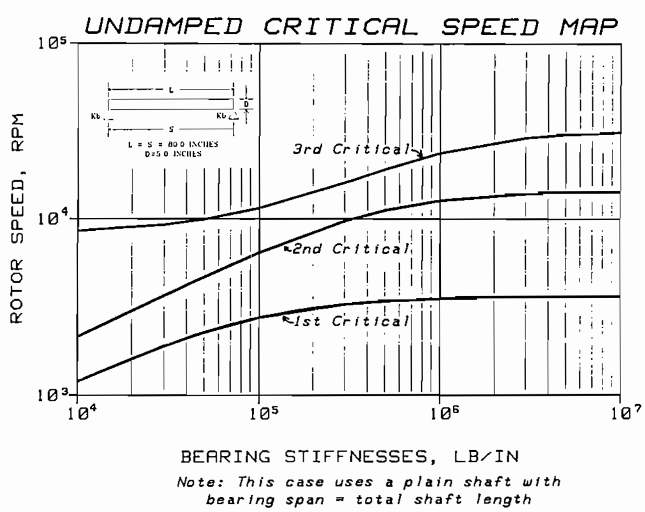

Figure 8. Example Rotor—Uniform Shaft Showing Critical Speed Variation As Support Stiffness Is Varied.

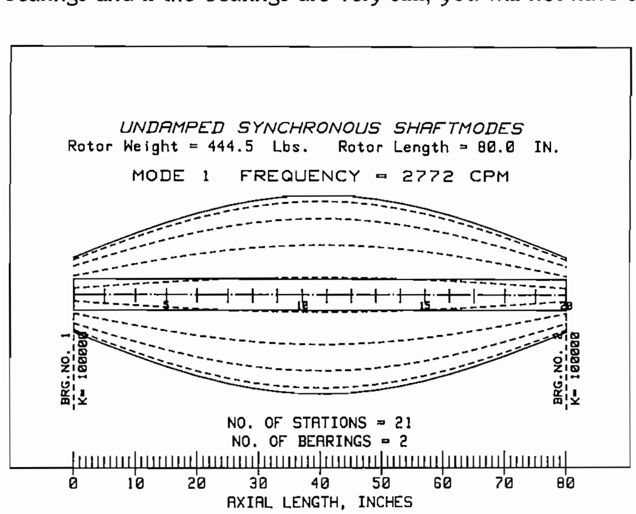

Figure 9a. Example Rotor—Uniform Shaft Showing Typical Modeshaped for the Case of Very Flexible Bearings.

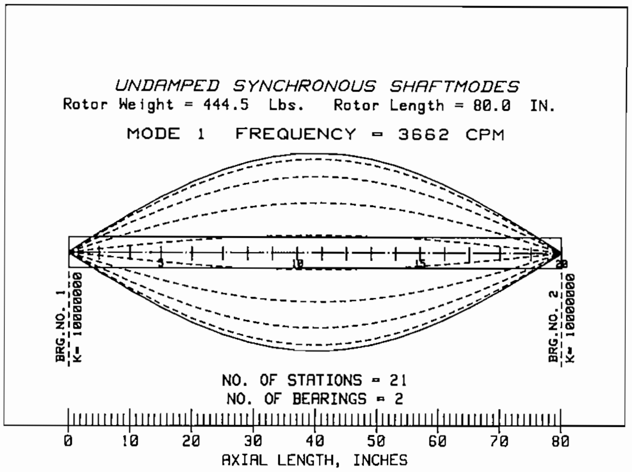

Figure 9b. Example Rotor—Uniform Shaft Showing Typical Modeshapes for the Case of Extremely Stiff Bearings.

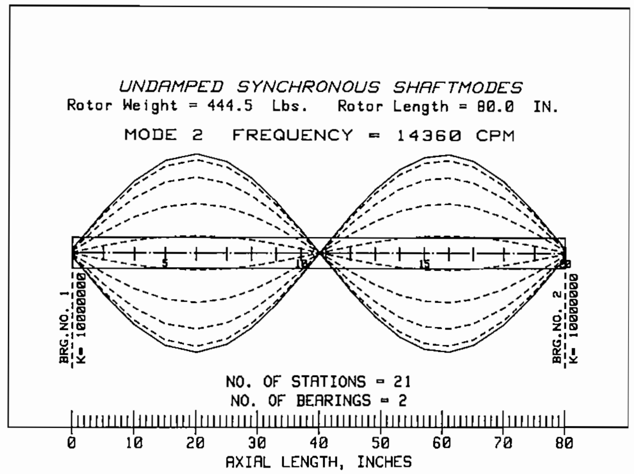

Figure 10b. Example Rotor—Uniform Shaft Showing Typical Modeshapes for the Case of Extremely Stiff Bearings.

good picture of the rotor's true motion elsewhere (like those labyrinth seals being wiped out at midspan). With softer bearings, the vibration probe is giving a truer indication of overall rotor amplitudes at the first critical speed.

To explain the other reasons, let's bring bearing damping into the picture. What does damping do for you and your machinery? Damping is the *only* mechanism that can dissipate critical speed energy. In vectorial terms, at the critical speed, the stiffness and inertia terms are 90 degrees out-of-phase with the rotor's motion, leaving only the damping to counteract the critical speed amplitude. Remember, from the basic equation of motion, that damping is proportional to velocity. So for damping to be effective, there must be motion in the bearing area. Thus, if your bearings are very stiff and "clamp down" on the rotor motion, the mechanism for effective damping has disappeared. Now, your probes not only don't "see" anything at the first critical speed, but the lack of damping generation within the bearings results in larger amplitudes elsewhere on the rotor. Do not get the idea that opening up the clearances in your machine's bearings to get more damping is necessarily a good idea. Again, the entire system must be evaluated.

The modeshapes for soft and stiff bearings for the second critical speed of our example rotor are shown in figures 10a and 10b. For flexible supports, Figure 10a, the modeshape is *pivotal* in nature, with a node point in the center-span, large amplitudes at the bearings and little rotor bending. The stiff bearing case, Figure 10b, shows that a lot of rotor bending has been introduced and, while there is still a node point at center-span, there are now two maximum amplitude areas at about the quarter-span points. Again, node points appear near the bearings as they "clamp down" on rotor motion and, again, the damping will be reduced. A well designed rotor-bearing system should not have much problem with the second critical, if the bearings can effectively contain the rotor motion and effectively dissipate the critical speed's energy, since the mid-span amplitudes will always be lower than the amplitude at the bearings. Indeed, this author has seen machines that run through or near their second critical speed continuously and run quite well. These machines have fairly compliant supports and as long as the bearings can contain the rotor unbalance forces, the second critical speed is virtually suppressed. This is also why many people criticize the term "critical speed," since they aren't always "critical."

The soft and hard bearing modeshapes for the third critical speed are shown in Figures 11a through 11c. The first figure shows the case of zero support stiffness and one can see from this why this is called the first bending critical. The rotor's bearings have minimal effect (compare Figures 11a and 11b)

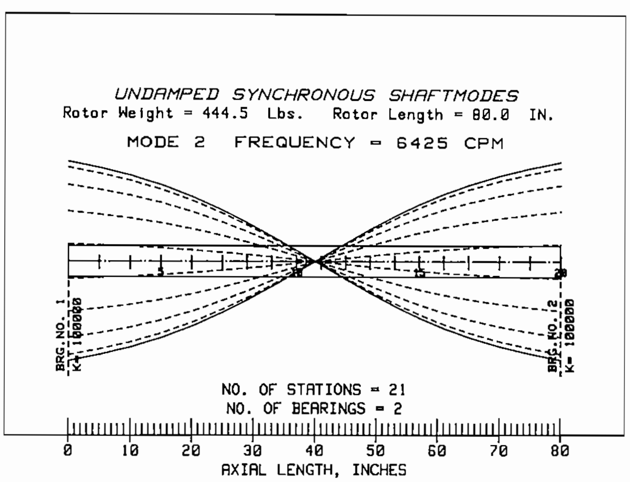

Figure 10a. Example Rotor—Uniform Shaft Showing Tpical Modeshapes for the Case of Very Flexible Bearings.

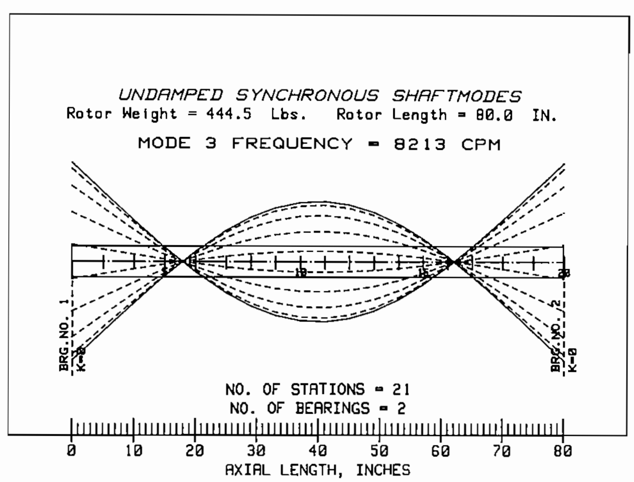

Figure 11a. Example Rotor—Uniform Shaft Showing Third Mode Modeshape for the Case of Zero Support Stiffness.

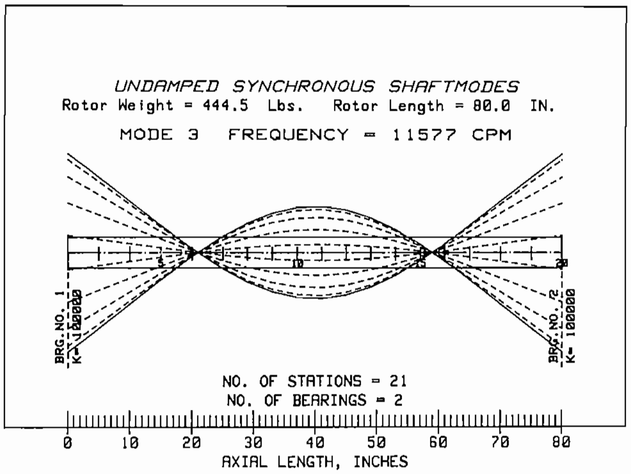

Figure 11b. Example Rotor—Uniform Shaft Showing Typical Modeshapes for the Case of Very Flexible Bearings.

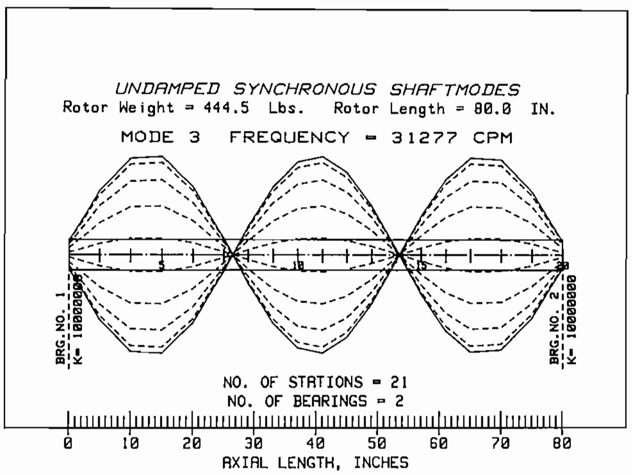

Figure 11c. Example Rotor—Uniform Shaft Showing Typical Modeshapes for the Case of Extremely Stiff Bearings.

on the modeshape until they become very stiff and force nodal points at the bearings (Figure 11c). As the bearings become very stiff, one can see that reverse bending occurs on the rotor, which can be very dangerous. This is the reason it is so important to avoid bending mode critical speeds. Higher modes are present in a few special cases, but will not be discussed here except to say that they are all bending mode critical speeds.

Now, having seen the effects of support stiffness on the first three critical speeds, look at how other parameters, such as bearing span and shaft diameter affect the critical speeds.

The Effect of Bearing Span —Between Bearing Systems

Logically, one thinks that a long span between bearing supports will make the system more flexible and, thus, lower the critical speeds. Indeed, since the shaft stiffness equation has the length term cubed and the first critical speed equation takes the square root of this, then the first critical speed should vary by the $3/2$ power with bearing span and indeed the variation approximates this. However, support stiffness must be taken into account as well. How then does bearing span affect the second and third critical speeds? The answer is not obvious. Look at what happens if we decrease the bearing span of our example rotor by 25 percent, from 80 inches to 60 inches. The new stiffness is:

$$K_s = [48 \times (30 \times 10^6) \times 30.68]/60^3$$
$$= 205{,}000 \text{ lb/in}$$

Compared to the 80 inch span case stiffness of 86,286 lb/in, this is an *increase* of 137 percent in shaft stiffness. Refer to Figure 12 to see how this span change affects the first three critical speeds. The solid lines are the 80 inch span case and are identical to Figure 8. The dashed lines are the critical speeds for the 60 inch span case. Looking at the first critical speed, one can see that there is an overall increase in the first critical speed, regardless of support stiffness, but since the mode is very translational at very low support stiffnesses, the bearings exert less influence than at higher support stiffnesses when bending is introduced. The shorter span induces more bending at high support stiffnesses and, thus, drives up the first critical speed. Above 1,000,000 lb/in, the increase is a constant 70 percent.

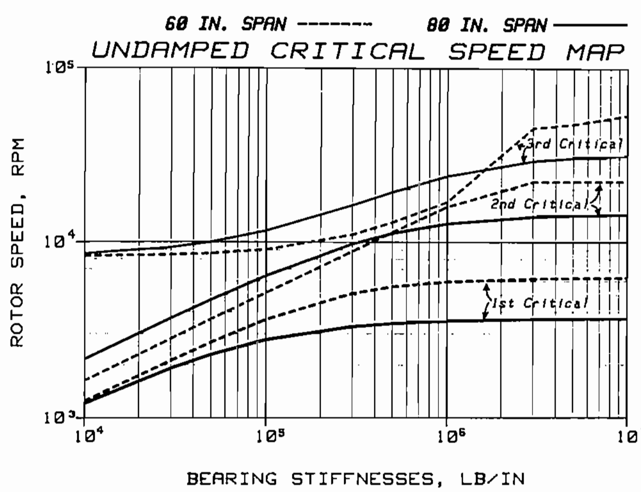

Figure 12. Example Rotor—Uniform Shaft Showing Critical Speed Variation As Support Stiffness Is Varied for Two Different Bearing Spans—80 Inches and 60 Inches.

The second critical speed behaves somewhat differently. At low stiffnesses, the shortened bearing span causes a decrease in the second critical speed; the two curves cross at about 500,000 lb/in support stiffness. Then, for higher stiffnesses, the second critical speed is higher for the shortened span case. We must examine the modeshapes to explain this case and the third critical speed case, which behaves similarly to the second critical speed case, experiencing a decrease in frequency at low stiffnesses and an increase at very high stiffnesses.

The first critical speed modeshapes for low and high stiffness with the 60 inch bearing span are depicted in Figures 13a and 13b. At low stiffness, the almost cylindrical modeshape does not depend as much upon where the bearings are, since the amplitude along the rotor is almost the same everywhere and the rotor is able to "float" around in the available bearing clearance. However, as the bearings begin to exert more influence, they cause more bending in the shaft until, as in Figure 13b, the nodal points are drawn inward to the bearing centerlines. Thus, the first critical is increasingly driven upward by the increased resistance to bending.

The second critical speed modeshapes for low and high stiffness for the 60 inch bearing span are shown in Figures 14a and 14b. At low stiffnesses, this mode is pivotal with maximum

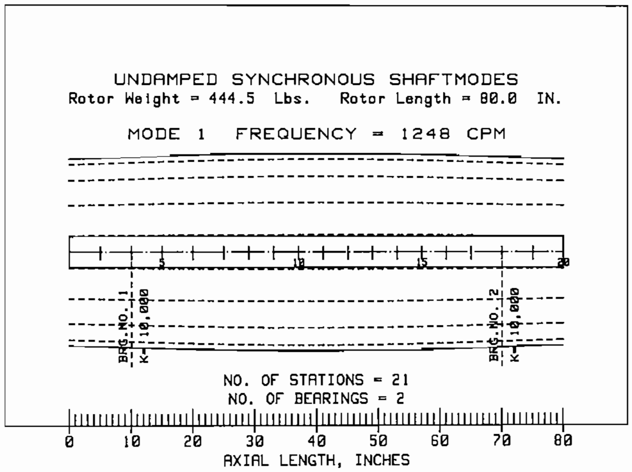

Figure 13a. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Flexible Bearings.

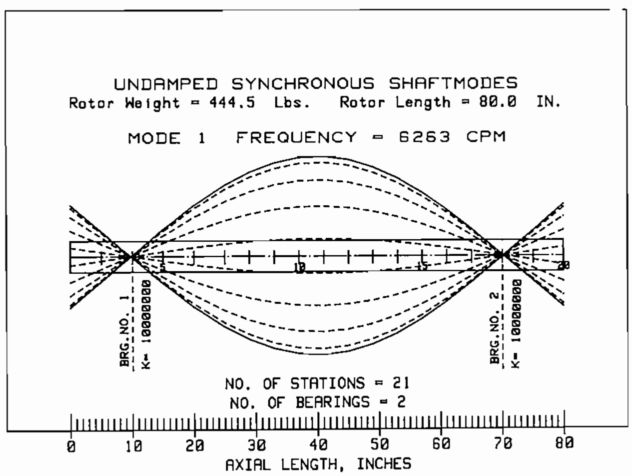

Figure 13b. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Stiff Bearings.

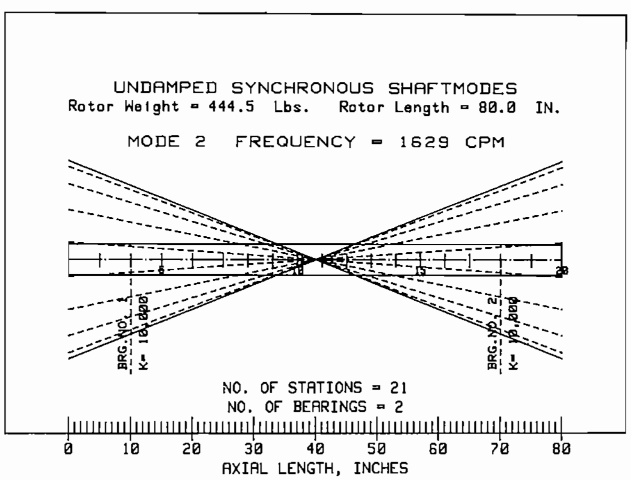

Figure 14a. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Flexible Bearings.

amplitudes at the rotor ends and a node point in the center. Moving the bearings *inward* moves them away from the area of maximum amplitude and thus decreases their effectiveness. The closer to the pivot point (node), the lower the second critical speed will be at low suppon stiffnesses. As with the first mode, as the bearings "clamp down" on the shaft, increased bending is induced and the shortened span increases the resistance to bending, driving up the second critical speed. At very high stiffnesses, there are now three node points *and* a stress reversal in the shaft at the second critical speed.

The third critical speed experiences a similar phenomena as shown in Figures 15a and 15b. At very low support stiffnesses the span will have no effect whatsoever; but as the lower support stiffness begins to affect the third mode, the bearings are now closer to the node points and, thus, have a reduced effect, lowering the third critical speed's frequency. As the bearing stiffnesses increase, more and more bending is induced as the bearings try to clamp down on the shaft. In this case, there are two stress reversals on the shaft and a great deal of shaft strain energy.

Thus, changes in bearing span are dependent upon the system's support stiffness. The first critical speed can be effectively raised by a decrease in span and this method is often used

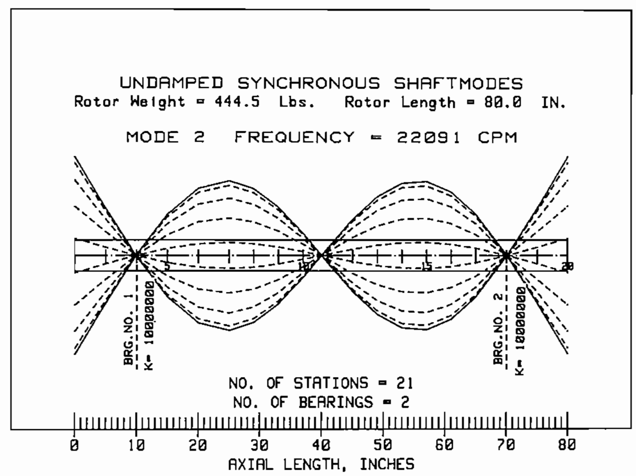

Figure 14b. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Stiff Bearings.

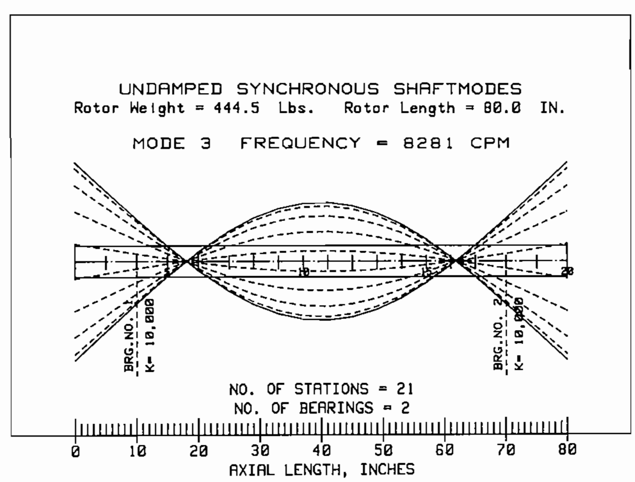

Figure 15a. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Flexible Bearings.

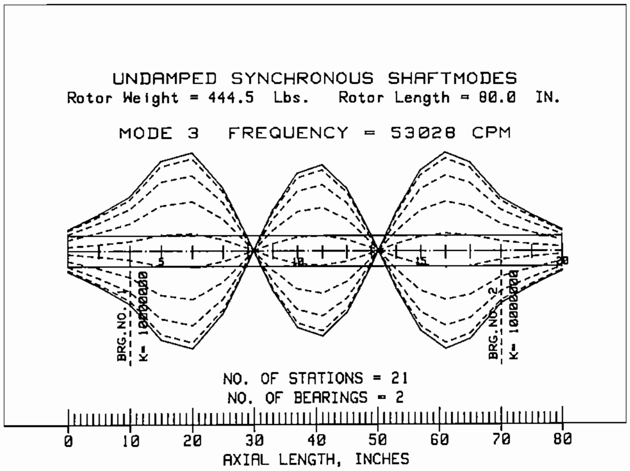

Figure 15b. Example Rotor—Uniform Shaft Showing Critical Speed Modeshapes for the Case of Very Stiff Bearings.

to raise the first critical speed out of an operating range. It will also raise the second critical, if there is sufficient support stiffness. One would hope that this would not have to be done to influence the third mode.

Changes in bearing span must also take into account the location of the rotor's node points. Bearings that are near node points are less effective than those at high amplitude points. This is why the second critical speed is lowered, as the bearings are moved inward toward the central node point.

The Effect of Shaft Diameter —Between Bearing Systems

The third major factor which controls critical speeds is shaft diameter. Designers of centrifugal compressors like small shaft diameters, so that they can make the impeller eyes small, bearings small, seals small and the overall design compact. Since shaft diameter affects the moment of inertia, I, then the first critical should vary by the square root of I. Let's take our example rotor, hold the span constant at 80 inches and vary the shaft diameter by first decreasing it 25 percent to 3.75 inches and then increasing it 25 percent to 6.25 inches. Total rotor weight will vary as the diameter changes (total weight is a function of radius squared) and you will see that the weight changes have an effect as well as the shaft diameter changes. Again, we must look at the effect as a function of support stiffness. The critical speed map which compares shaft diameters as a function of support stiffness is shown in Figure 16. The solid line is again the baseline case of 5 inch diameter. The dashed line is the smaller 3.75 inch shaft and the dash-dot-dot line is the larger 6.25 inch case.

The first critical speed lines show that, at low support stiffnesses, the smaller diameter shaft has a *higher* first critical speed and the larger shaft has a *lower* first critical speed. This is because the dominant factor with very low support stiffness is the rotor weight. Since the first critical is $\omega_n = \sqrt{K/M}$, the lighter rotor will raise the first critical speed and the heavier rotor will decrease it. As stated before, since there is almost no bending at low support stiffnesses, the system will be affected more by other factors (in this case mass) than by the system stiffness. As support stiffness rises, the logical change occurs and the smaller shaft has a much lower first critical speed. Since the smaller diameter shaft is more flexible, the bearing stiffness becomes dominant faster. Notice how this curve "flattens out" much faster and approaches an asymptotic value. This more flexible shaft has a much smaller *bearing dependent* region on the critical speed map. The stiffer shaft increases the first critical speed at moderate and high support stiffnesses and also does not "flatten out" as fast. The stiffer shaft has a larger bearing dependent region on the map.

The second critical speed curves show a response similar to the first critical and for the same reasons. The mass influence is even greater and the support stiffness must be greater than about 750,000 lb/in before the larger shaft's second critical increases over the baseline case.

The third critical is a bit different in that at low support stiffnesses, the system is essentially "bearingless" and the stiffer shaft will be more resistant to bending and will have a higher third critical speed. Likewise, the smaller diameter shaft will be less resistant to bending. In the middle stiffness range, there is a minimal effect, and at high stiffnesses, the increased induced bending meets more resistance from the stiffer shaft and, thus, raises the third critical speed.

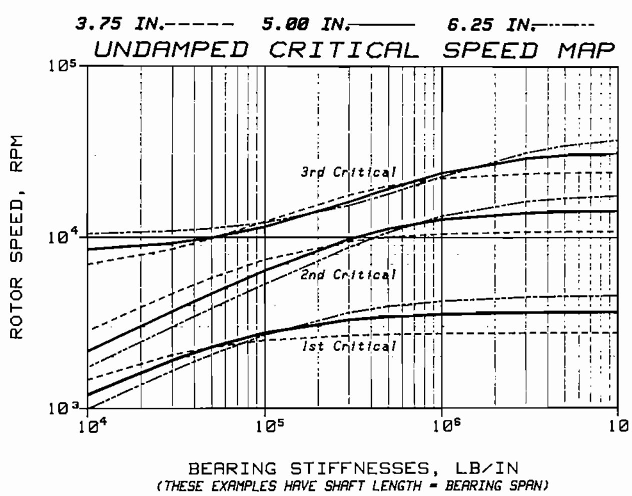

Figure 16. Example Rotor—Uniform Shaft Showing Critical Speed Variation as Support Stiffness Is Varied for a 25 Percent Change in Shaft Diameter. 3.75 Inches, 5 Inches, 6.25 Inches.

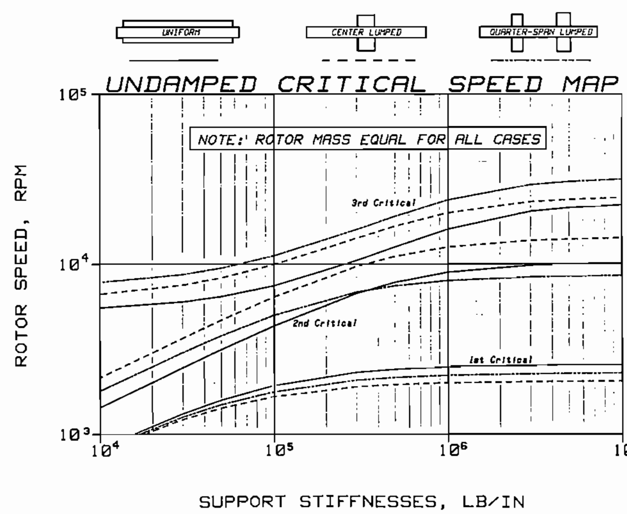

Figure 17. Example Showing the Effect of Mass Distribution on a Uniform Rotor with Shaft Length-Bearing Span.

The Effect of Mass Distribution —Between Bearing Systems

The fourth effect on critical speeds is the placement of the concentrated masses on the shaft. If we limit ourselves to between bearing rotors (overhung rotors will be covered later), the limiting cases would be the uniform distribution case, such as a multi-wheel compressor, with the stages uniformly distributed and the case of a single mass lumped in the center of the span, such as a single wheel turbine. Another interesting case would be to have two-wheels with half the central mass lumped at the quarter-span points. The critical speed map for this, as we take our example rotor and distribute 500 pounds as outlined above, is shown in Figure 17. The solid line is the uniform distribution case and the dashed line is the center-lumped case. In all cases, the total rotor mass is identical, as is the shaft diameter of 5 inches and the bearing span of 80 inches. Again, we find that the effects are dependent on support stiffness.

The first critical speed responds by having the highest critical speed for the uniform distribution and the lowest for the center-lumped case. This is logical, when you think of the modeshape of the first critical speed. The maximum amplitude is at the rotor center and, thus, the concentrated mass more easily affects the first critical. As the mass is distributed outward, the mass near the bearings contributes less to the *modal* mass and, thus, the first critical speed is raised.

The mass distribution has an almost opposite effect on the second critical speed. Regardless of stiffness, the center lumped case has the highest second critical speed of the three cases. This is because the node point is in the center of the span and this lumped mass cannot exert much influence at all. At low support stiffnesses, the quarter span case has the masses close to the nodal point and their effectis minimal, driving up the second critical speed. However, as the stiffness increases, the maximum amplitudes move toward the quarter-span points (Figure 10b) and the masses are then very effective and *lower* the second critical speed.

The third critical speed is again different. The uniform case has the lowest third critical speed and the quarter span case has the highest. The modeshapes provide the explanation. The quarter-span case has the masses close to the node points and, thus are ineffective, causing an increase in the third critical speed. The center span case puts the mass at a point of maximum amplitude points and is most effective in lowering the third critical speed.

To summarize the effect of mass distribution, mass is effective in *lowering* critical speeds when it is near a maximum amplitude point. The *modal mass* for that mode is increased, causing a decrease in the critical speed of that mode. To drive up a critical speed, put the concentrated mass at a node point where it will not increase the effective modal mass.

OVERHUNG ROTORS AND CENTRIFUGAL PUMPS

This is really a special case of mass distribution and bearing span change. To examine these effects, this example will take our original example, move one bearing inboard 20 inches and hang a 500 pound mass on the end. A cross section of this new rotor is presented in Figure 18. This will be compared to the 80 inch span between-bearings case with a 500 pound mass lumped in the center of the span. Other comparisons could be made, but this one will illustrate the necessary points. Total rotor length and weight will be the same, mass placement and bearing span will differ.

The critical speed map comparing these two cases is shown in Figure 19. The solid line is the overhung case and the dashed line is the between-bearing case. At low bearing stiffnesses, the

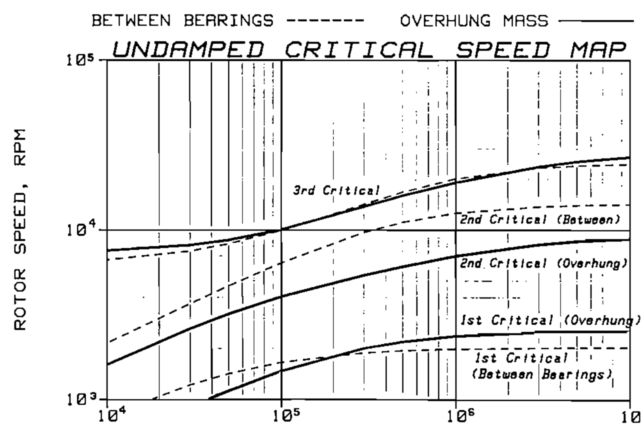

Figure 19. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Comparing Overhung with between Bearings Rotors.

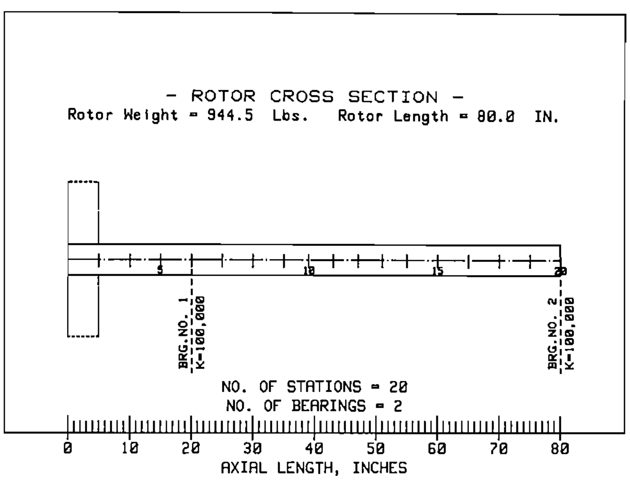

Figure 18. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

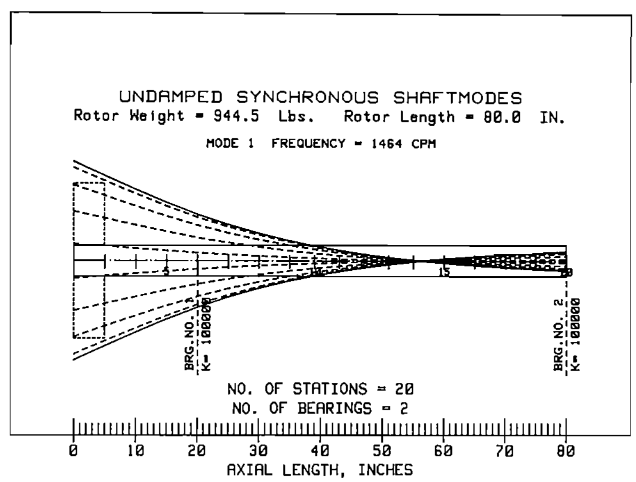

Figure 20a. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

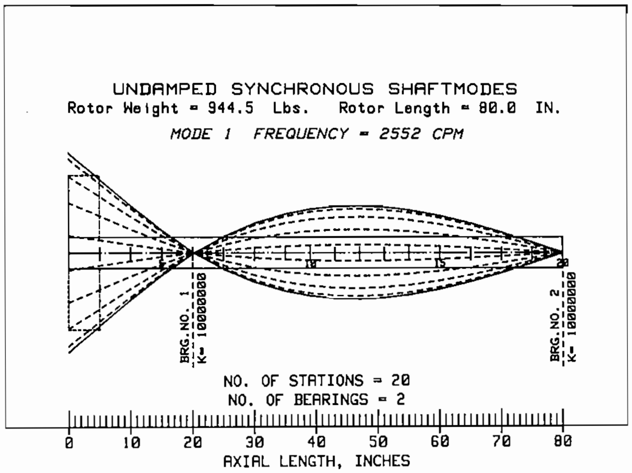

Figure 20b. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

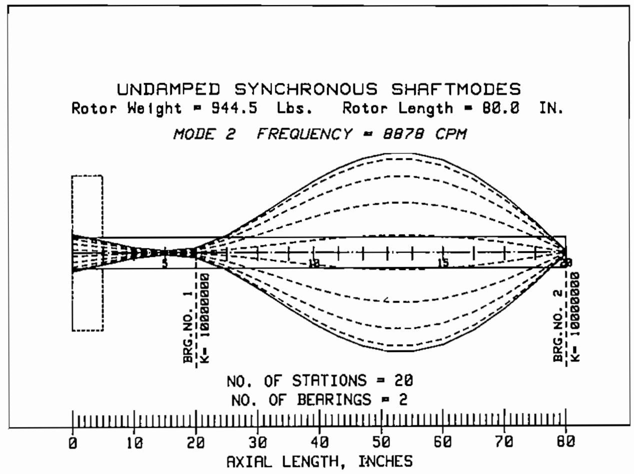

Figure 21b. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

overhung rotor has a much lower first critical speed and at high stiffnesses the between-bearing rotor has the lower first critical speed. The explanation lies in the modeshapes again. The overhung modeshapes for flexible and stiffbearings are shown in Figures 20a and 20b, respectively. At low support stiffnesses, the first mode is pivotal, with maximum amplitude at the concentrated mass and a node point at about midspan. The distance from the node to the maximum amplitude is greater than in the between-bearings case (Figure 14a) and, therefore, the system is effectively more flexible, lowering the first critical speed. As soon as the bearing stiffness increases above 250,000 lb/in, the node point for the overhung rotor moves toward the bearing near the mass and the distance gets smaller, like shortening the span, and the first critical is driven up.

The overhung rotor always has a lower second critical and the modeshapes presented in Figures 21a and 21b show why. The second modes for both cases have the node near the concentrated mass for both stiffness cases, thus giving the mass minimal effect. The overhung case, however, has a much greater distance between the node and the far end of the rotor making the shaft's effective modal mass greater and, thus, lowering the second critical speed.

The third mode is not greatly effected speed-wise by the overhung effect, but as can be seen in Figures 22a and 22b, the modeshapes are different from Figures 11b and 11c. The bearing near the overhung mass is least effective since it is inherently near a node point for the third critical speed.

One should note that in the design of an overhung machine, there is usually a coupling to the driver on the opposite end from the lumped mass. This creates somewhat of a double-overhung effect and makes the first mode entirely pivotal. Thus, the coupling vibration, while below or near the first critical speed, will be 180 degrees out of phase with the wheel end vibration. This is *very important* when balancing an overhung machine. The other important point is that the design of the two bearings is quite different. Usually, the wheel-end bearing is carrying the majority of the load while *the coupling end bearing may be very lightly loaded or even negatively loaded.* The design of this bearing then becomes critical from a load carrying standpoint and, especially, from a stability standpoint.

To illustrate what a typical API type process pump's dry critical speeds look like, an actual pump was modelled and the shaft cross section is shown in Figure 23. This rotor is supported

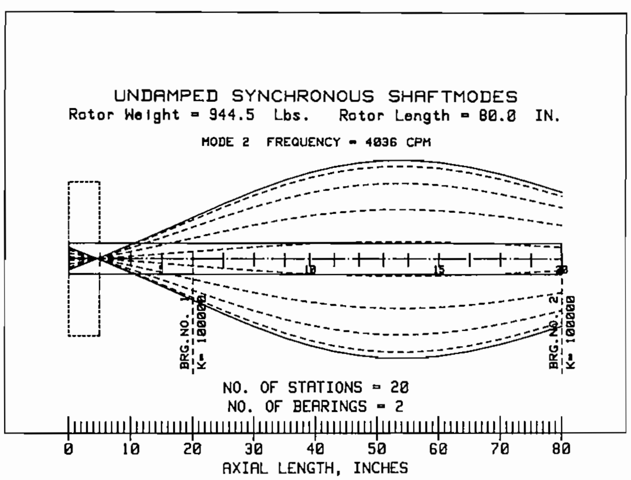

Figure 21a. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

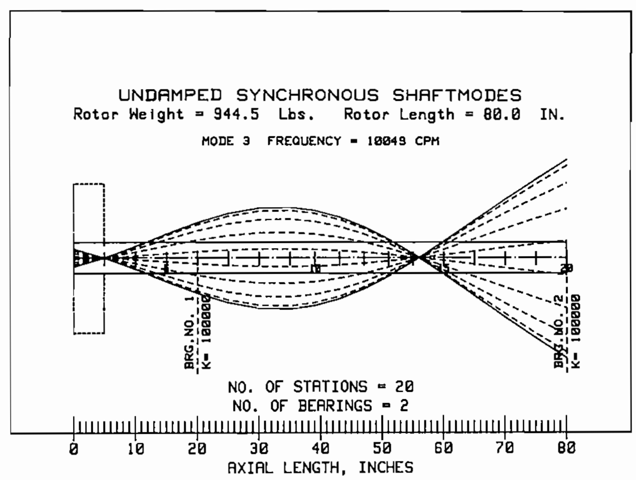

Figure 22a. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

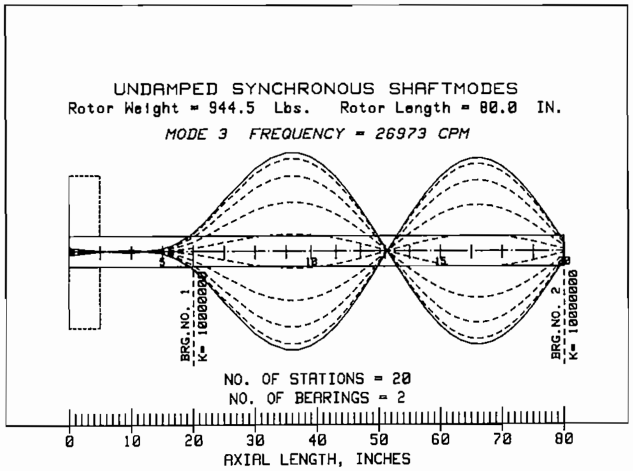

Figure 22b. Overhung Rotor Example—Uniform 5 Inch Shaft with 500 lb External Mass, Examining Single Overhung Modeshapes and Critical Speeds.

in rolling element bearings and is run at 3550 rpm. When in good condition and properly fit, rolling element bearings are very stiff. However, the governing factor for cases such as this usually becomes foundation flexibility, so that support stiffnesses greater than two to three million lb/in are not realistic. Often the pump designer counts on the wear ring and bushing close clearances to provide the needed stiffness and damping to control rotor motion and, as long as these clearances do not open up due to wear or corrosion, the design works.

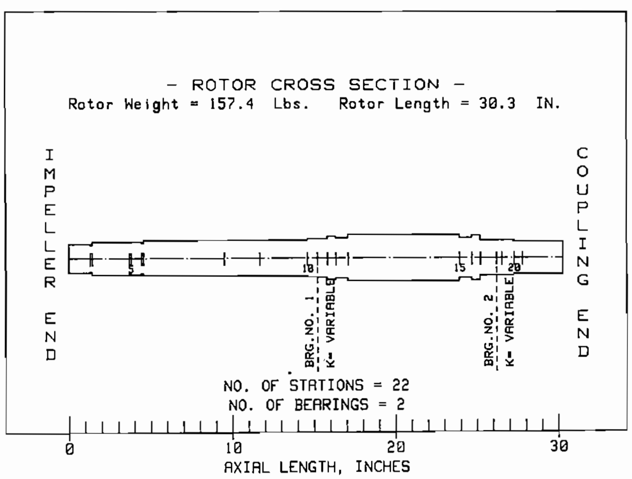

Figure 23. Single Stage Overhung API Refinery Type Process Pump. Typical Pump Example to Show Critical Speeds and Modes.

The undamped critical speed map for this pump (Figure 24) shows that the support stiffness must be above 1,000,000 lb/in to provide sufficient margin of separation between the first critical speed and operating speed. Indeed, as long as bearing fits and wear ring clearances are maintained, this unit runs quite well. The modeshape of this rotor, with support stiffnesses at 1,000,000 lb/in, is shown in Figure 25. Note that it is a classic pivotal mode with little amplitude at the bearings and maximum amplitude at the impeller eye. From this, one can see that if this mode is encroached upon, the wear ring will be damaged first, lowering the first critical speed even further. The second mode, shown in Figure 26, has two node points, again with minimal

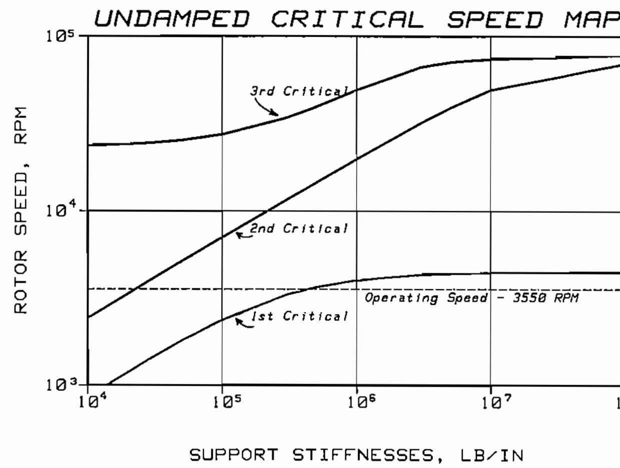

Figure 24. Single Stage Overhung API Refinery Type Process Pump. Typical Pump Example to Show Critical Speeds and Modes 8×6×13—3550 RPM—Thrust BRG. 7311DB; Radial BRG. 5212.

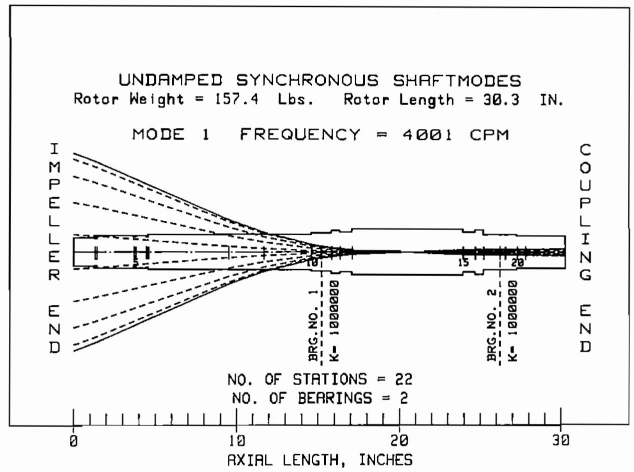

Figure 25. Single Stage Overhung API Refinery Type Process Pump. Typical Pump Example to Show Critical Speeds and Modes.

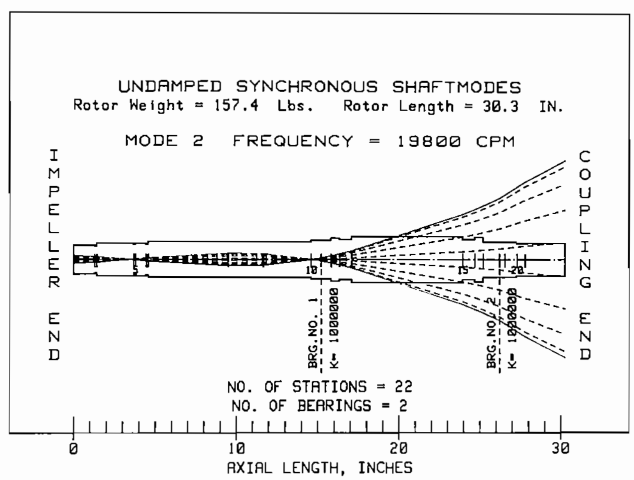

Figure 26. Single Stage Overhung API Refinery Type Process Pump. Typical Pump Example to Show Critical Speeds and Modes.

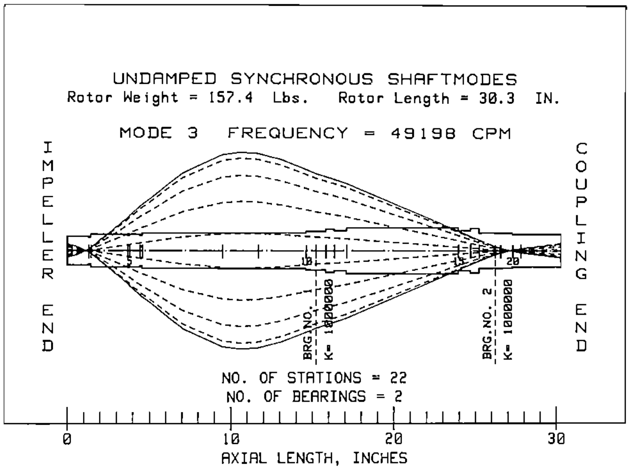

Figure 27. Single Stage Overhung API Refinery Type Process Pump. Typical Pump Example to Show Critical Speeds and Modes.

amplitude at the bearings and impeller and maximum amplitude at the coupling. This mode is sufficiently high so that it is not normally a problem unless vane-pass excitation (i.e., 5 vanes times 3550 rpm = 17750 cpm) causes it to be excited. Larger coupling masses will lower this mode and there can be resonant excitation, particularly with poor coupling balance. The third mode (Figure 27) is a typical bearingless free-free mode with lots of shaft bending. Chances of encountering this mode in a pump are virtually nonexistant.

Summary of Geometry Conditions

This section has presented, in as general terms as possible, the effects of geometry on critical speeds and the various elements that go into these effects. Briefly these effects are:

1. *Support stiffness.* This is the most influential factor in determining critical speeds. Other factors must be looked at as a function of support stiffness to completely examine their effects. In general, the higher the support stiffness, the higher the critical speed. The critical speed map can be divided into three areas (Figure 8). At low support stiffnesses (relative to shaft stiffness) the critical speeds of a system are almost entirely dependent upon the stiffness of the bearings (sloped part of the curve). When support and shaft stiffnesses are comparable, both the shaft geometry and bearing stiffness contribute to critical speed determination. When the supports become very stiff, the system is said to be shaft dependent, because bearing changes cannot raise the critical speeds significantly, only shaft geometrical changes can.

2. *Bearing span.* Small changes in bearing span can significantly alter the frequency of critical speeds. Detailed analysis is required to determine the effects on a given rotor system. This is a difficult retrofit problem.

3. *Shaft Diameter.* Stiffer shafts mean higher critical speeds, if the bearing stiffness is sufficient. Otherwise, the extra mass counteracts the increased stiffness and may yield only minor improvements in critical speed frequency. This is a very difficult retrofit problem requiring shaft, wheel, and case modifications.

4. *Mass placement.* Critical speed modification due to mass placement is almost entirely a function of modeshape. When masses are placed at maximum amplitude points, the effective modal mass increases and lowers that critical speed. Masses near node points have minimal effect on that mode.

5. *Overhung mass rotors.* The critical speeds of overhung rotors are very dependent upon support stiffness. Most of the amplitude is at the overhung mass for the first critical speed and the mass becomes a node point for the second and third critical speeds. Pumps often rely on the added stiffness of wear rings and close clearance bushings to avoid the first critical speed. Excessive wear can minimize this stiffening ability and cause the pump to encroach on the first critical speed. Due to the presence of fluid forces in a pump, the dry critical is only a starting point for rotordynamic analysis. Rotor-structure interaction should also be considered as well as the possible excitation of resonances by vane-pass pulsations. Usually rotors are double overhung, due to the presence of a coupling, and the proper design of the bearings for this type of machine is crucial. Very heavy couplings will aggravate the critical speed problem.

By now, it is easy to see that the analysis of rotor-bearing systems is quite complicated and, in many cases, the combination of several factors may yield non-obvious results. There are other complications involved in analyzing a rotor-bearing system: The response of a rotor system to unbalances (a measure of the rotor's sensitivity to deterioration or deposit buildup) is vital to long-term reliability. The system's stability must be determined if the design is at all doubtful. Thus, to form a total picture of a rotor design, the design auditor must resort to advanced computerized rotordynamics analysis. There are many other factors which complicate the analysis of pumps, mainly the fluid forces, and these must be considered. Finally, support structure interaction may require the use of very sophisticated finite element design tools, which are far too complex to be covered here.

NOMENCLATURE

C damping, lbf-sec/in
C_c critical damping, lbf-sec/in
E Young's modulus, psi
I area moment of inertia, in^4
K spring stiffness, lbf/in
K_b bearing stiffness, lbf/in
K_s shaft stiffness, lbf/in
M mass, lbf/in (mass = weight/g)
N rotative speed, rpm
Ncx critical speed number x, rpm
pi 3.14159
S Sommerfeld number
ω frequency
ω_d damped natural frequency, rad/sec
ω_n undamped natural frequency, rad/sec
x displacement, inches
$\dot{x}$ velocity, inches/second^2
$\ddot{x}$ acceleration, inches/second^2
δ logarithmic decrement, dimensionless
§ Damping ratio—C/C_c, dimensionless
λ real damping eigenvalue, sec^{-1}

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Dr. Chamieh is currently working with Byron Jackson, where he is involved in the design and testing of hydraulic turbines as well as in various aspects of recirculation in pumps. Prior to joining Byron Jackson, he participated in the design and construction of the Rotor Pump Test Facility located at Cal Tech and intended for the study of the dynamic behavior of the whirl of centrifugal turbomachine rotors. He also developed analytic models for internal impeller-volute systems.

Dr. Chamieh is an associate member of ASME and a life member of Tau Beta Pi and Pi Tau Sigma. He is the author or co-author of more than nine papers.