INTEGRATION OF OIL SANDS TAILINGS AND RECLAMATION PLANNING WITH LONG-TERM MINE PLANNING

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ABSTRACT

An optimized long-term mine planning determines the best order of material extraction, so that the Net Present Value (NPV) becomes maximum. In the case of oil sands surface mining, one of the key constraints in NPV maximization is the volume of tailings slurry that will be generated downstream. Specifications of tailings slurry are also important from environmental perspective. Moreover, most of the material that will be used for the reclamation of tailings ponds in later periods is generated in extraction and processing of oil sands. In this paper, an integrated optimization framework is proposed to maximize the NPV of the produced oil sands, with respect to tailings capacity constraints and reclamation material requirements. A mixed integer linear programming (MILP) model is developed to find the optimal solution for long-term mine planning problem. The proposed model is coded in Matlab and has been run using CPLEX for verification. The results for real-case oil sands dataset show that the optimal production schedule meets material requirements for reclamation, tailings volume is within tailings capacity range in all periods and the production schedule follows the predetermined horizontal direction.

KEYWORDS

Mine planning, reclamation, tailings, mathematical programming, oil sands

INTRODUCTION

Open-pit mining is used to extract near-surface oil sands reserves in Northern Alberta. A mine plan determines the best sequence of block extraction, the production schedule, in such a way to maximize the NPV over the mine life. There are a number of downstream consequences associated with any production schedule, such as the volume of tailings produced downstream as the result of bitumen purification process. The extracted material is sent to the processing plant for further processing of bitumen. There are environmental consequences associated with bitumen processing (Rodriguez, 2007; Singh, 2008; Woynillowicz & Severson-Baker, 2009). Typically, tailings management is considered separately from mine planning. In most of current practices, when the optimal mine plan is developed, the volume of tailings is calculated and further decisions are made about sending the slurry to different available tailings facilities. The volume and specifications of tailings is closely connected to the mining operation and processing of the oil sands. Therefore, it is reasonable to include tailings management considerations, such as tailings volume, as a set of constraints in the mine planning optimization model.

Oil sands operators must reclaim tailings ponds before leaving the mine site. Most of the material that is used for reclamation comes from mining operations, such as over burden, oil sands processing, or tailings coarse sand. When the mining project approaches to the mine closure, the reclamation phase is planned in a way to make sure that the essential material for capping of tailings ponds is available. It shows the connection between mining operations and reclamation material requirement. Therefore, reclamation material requirement can also be integrated into the mine planning model.

Long-term and short-term mine planning models maximize the NPV in strategic and operational levels, respectively. In the literature, mine planning models for different time horizons are well introduced (Askari-Nasab, Pourrahimian, Ben-Awuah, & Kalantari, 2011; Askari-Nasab, Tabesh, & Badiozamani, 2010; Ben-Awuah & Askari-Nasab, 2011; Ben-Awuah, Askari-Nasab, & Awuah-offei, 2012). Mine waste stream management and dyke construction for tailings dams is also included in mine planning models (Odell, 2004; Rodriguez, 2007). There has also been research addressing environmental issues, including reclamation costs in mine design(McFadyen, 2008). However, tailings management and reclamation plans are not integrated in typical oil sands mine planning models and the merger between these three research areas is the missing part in the literature. In this paper, the objective is to develop an integrated long-term mine planning framework that maximizes the NPV and minimizes the reclamation material handling costs at the same time. The integrated model includes tailings capacity constraints and material requirements for tailings pond reclamation.

PROBLEM DEFINITION

To find the optimal production schedule that meets the tailings and reclamation requirements, an integrated mine planning model must be developed. The inputs to this model are: (1) the optimal pit limits that based on Lerch Grossman algorithm, determine which blocks are economically extractable, (2) the block data, including spatial coordination of the blocks and the grade of different elements in each block, and finally (3) a tailings model that determines the volume of tailings slurry, plus the volume of fine material, sand and water resulted from extraction and processing of each block.

A sample reclamation plan that is published to fulfill the requirements of Directive 074 (Shell-Canada, 2011) is a good example that shows the relation between the three concepts of long-term mine planning, tailings management and reclamation planning. Shell Canada considers dedicated disposal areas (DDA) for JackPine Mine (JPM) site at Athabasca river region in Alberta. Table 1 presents three main steps that are included in decommissioning of an external tailings facility, including construction, operations and closure. The time horizon in this decommissioning plan covers a period of 53 years, between years 2008 and 2061 (Badiozamani & Askari-Nasab, 2012a).

Table 1 - Summary of JPM decommissioning time line

Construction (2008 to 2011): • Preparation of Starter Dyke • Preparation of External Dyke Walls (centerline) • Preparation of Upstream Dyke
Operations (2010 to 2055): • TT deposition – initial filing period • Centrifuge Cake Manufacture / Deposition • TT deposition – in-pit tailings CST capping activities (2) • TT deposition – in-pit tailings CST capping activities • TT deposition – in-pit tailings CST capping activities • TFT transfer to SC1
Closure, Capping and Final Landform Design (up to 2055):

- Completion of TT deposition
- Trafficable tailings surface
- Overburden capping and drainage contouring reclamation cover soil placement
- Nurse crop coverage and cap settlement
- Re-vegetation
- Monitoring
- Completion of TT deposition

All of the three phases of construction, operations and closure are influenced by the long-term mine planning. Here are two instances that show how mine planning is important in decommissioning phases; (1) waste material is required for starter dyke construction in construction phase. The mine planning determines when and how much waste material is generated and can be used for reclamation. (2) Thickened tailings (TT) and coarse sand tailings (CST) are required for filling and capping in operations phase. Oil sands processing rate determines how much CST and TT is available in each period. The processing rate is directly related to the amount of mineralized material that is sent to the processing plant, determined in mine planning.

The reclamation plan proposed by Shell is just an example that shows the relation of mine planning and reclamation plan. In general, with any change in the mine plan, the amount of waste that is produced, as well as the volume of tailings slurry will be changed. Therefore, the reclamation plan must consider such changes accordingly to make sure that the required material will be ready in right periods. Knowing this fact, the objective of this paper is to develop an integrated strategic mine planning model that maximizes the NPV over the mine-life, with respect to the capacity of tailings facility and material requirement for tailings pond reclamation. In addition, a new term must be added to the objective function to minimize the cost of materials handling for reclamation purposes. The organizations of a typical model for mine planning (MILP-1) versus an integrated model (MILP-2) can be presented as:

MILP-1:

Maximize (NPV)
Subject to:

- *Processing plant constraints*
- *Mining capacity constraints*
- *Extraction precedence constraints*

MILP-2:

Maximize (NPV – reclamation costs)
Subject to:

- *Processing plant constraints*
- *Mining capacity constraints*
- *Extraction precedence constraints*
- *Tailings capacity constraints*
- *Reclamation material requirement constraints*

A schematic overview of integrated mine planning model is illustrated in Figure 1.

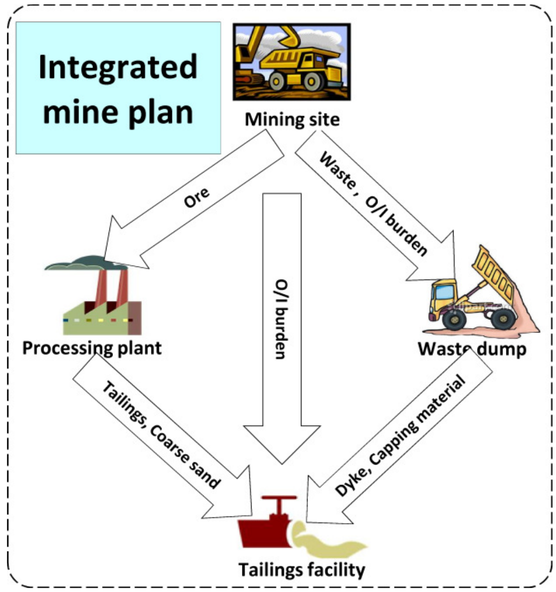

Figure 1 - An integrated mine planning, direct excerpt from (Badiozamani & Askari-Nasab, 2012a)

THEORETICAL FRAMEWORK

Directional Mining

A complete introduction of vertical precedence relation for the extraction of mining units (block or cuts) is presented in the literature (Askari-Nasab et al., 2011; Tabesh & Askari-Nasab, 2011). In a most complete version, vertical precedence is defined between each block and nine blocks on top of it. The nine blocks must be extracted first to get access to the block at the bottom. This is true for all blocks in a pit, except for the blocks in the first bench at the top. Figure 2a shows a schematic configuration of vertical precedence. Integer decision variables ($b_k^t \in \{0,1\}$) are defined to control the precedence between mining-cuts.

However, vertical precedence does not guarantee to have a feasible solution in many cases. For example in the case of oil sands surface mining, the mining operation must happen from one end of the pit and move forward in a specific direction so that the mined-out pit that is left behind becomes available as an in-pit tailings facility. Horizontal precedence must be defined between mining units to force the extraction progress in a predetermined horizontal direction. There are many numbers of horizontal directions. However, in this model only four main directions are considered as south-north, north-south, east-west and west-east (Figure 2b).

An example of horizontal precedence relation is presented schematically in Figure 2c. In this mining direction, three blocks of 1, 2 and 3 must be extracted to get access to the shaded block behind them. In a simplified version of horizontal precedence, only block 2 is assumed to be the horizontal predecessor for the shaded block behind (Figure 2d). It reduces the problem size and the implementation results show that this is a valid assumption.

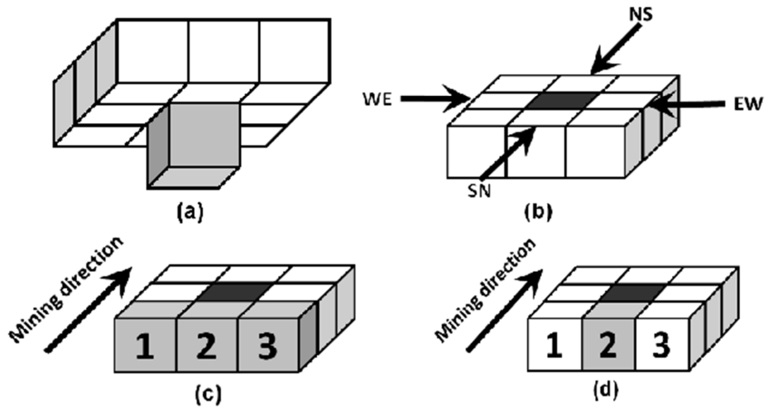

Figure 2 - Block precedence in vertical (a) and horizontal (b, c and d) directions, direct excerpt from (Badiozamani & Askari-Nasab, 2012b)

To make large-scale problems solvable, mining blocks must be aggregated into mining-cuts (Tabesh and Askari-Nasab, 2011). Mining-cuts do not have regular shapes and therefore, horizontal precedence cannot be defined between them in the same way that is defined for mining blocks. However, horizontal precedence relations are defined for mining-cuts based on the precedence relations between mining blocks within the cuts.

Pushbacks are the backbone of long-term mine planning. They are typically defined as the optimal pit limits (nested pits) resulted from increments in the revenue factors of the ore (bitumen). Horizontal precedence can be defined among pushbacks in different ways. In this paper, the following approach is used: if pushback 1 is predecessor of pushback 2 in a horizontal direction, then all the mining-cuts at the very bottom of pushback 1 are considered as the predecessors for all the cuts at the top of pushback 2. This will guarantee that extraction of mining-cuts in pushback 2 will not start before completion of extraction of all the mining-cuts in pushback 1.

The Mixed Integer Linear Programming (MILP) Model

Mixed integer linear programming is used to formulate the long-term mine production scheduling problem. Mining-cuts are used as the units of scheduling in MILP formulation framework (Tabesh & Askari-Nasab, 2011). Different sets of continuous variables are defined to facilitate extraction of ore portion, over/inter burden portion and tailings sand portion of each mining-cut and sending them to various destinations, while binary variables are defined to control extraction precedence between mining-cuts. The MILP has two objective functions, one for NPV maximization and the other one for cost minimization associated with reclamation materials handling. The overall profit from mining and processing of a mining-cut is proportional to the value of the cut and the total costs associated with mining, processing and materials handling cost for reclamation at a specified destination. The organization of the mathematical model is as follows:

Maximize

(NPV – reclamation material handling cost)

Subject to:

Mining capacity constraints,
Processing capacity constraints,
Reclamation material requirement constraints,
Total tailings capacity constraints,
Limits for fines, sand and water in tailings,
Quality of ore feed to the processing plant,
Mass balance relations for different portions of cuts,
Precedence constraints (vertical and horizontal)

Decision variables are defined in such a way that provides the required flexibility to the optimizer for sending different portions of material for processing, reclamation or dumping as waste. The concept of dynamic cut-off is used in construction of the MILP model, meaning that the optimizer determines the destination of each parcel in such a way to maximize the NPV, rather than having a fixed cut-off that predetermines material destination based on ore content of the parcels.

A complete formulation of the mathematical model including notation for sets, indices, parameters, decision variables and the MILP model is presented in the appendix.

CASE STUDY

To verify the proposed MILP model, an oil sands dataset is used, that includes 45,648 blocks with dimensions of 50 by 50 by 15 meters, in 9 benches. The mining blocks are aggregated into 478 larger units as mining-cuts. The whole mine site is divided into two pushbacks, separated by a river. It is assumed that pushback 1 proceeds pushback 2 in extraction. There are two destinations for the extracted material as: (1) the processing plant, and (2) the waste dump. The problem is solved for 12 periods. Matlab (MathWorksInc., 2011) is used for preparation of matrices for the objective function and the constraints. The Matlab code calls TOMLAB/CPLEX (Holmström, Göran, & Edvall, 2009), which solves the mixed integer linear programming model. The code is executed on a dual quad-core Dell Precision T7500 computer at 2.8 GHz, with 24GB of RAM. Some of the parameters for the case study are presented in Table 2.

Figure 3 shows the grade distribution in the pushbacks. High-grade areas are shown in dark colors (12% to 16%), while lighter patches represent low-grade material. To compare the NPV and decide the best mining direction, the model is run for two directions of East-West (E-W) and West-East (W-E). The Associated NPVs are compared in Table 3. The results show that mining in E-W direction generates a better NPV, which is almost 10% better than the optimal solution in W-E direction.

As Figure 3 shows, the right part of the mine site (East) contains higher grade material, comparing to the left (West). Therefore, it was expected to have a better NPV in E-W direction. Figures 4 and 5 present two sample plan views of resulted schedules for E-W and W-E directions, respectively.

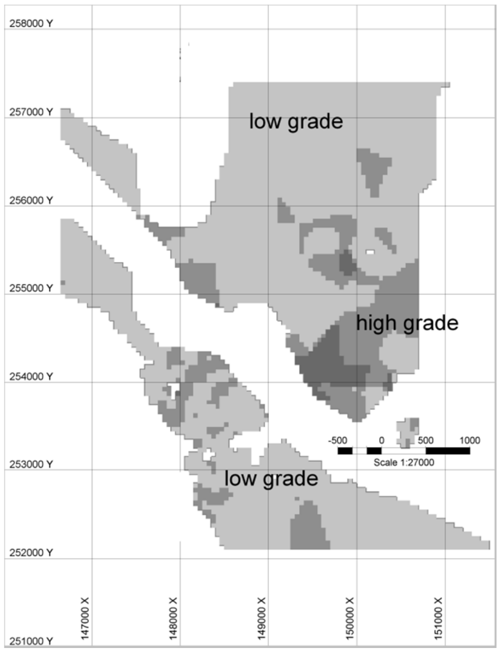

Figure 3 - Bitumen grade distribution (bench 4)

Table 2 - Parameters of the case study

Total tonnage of material	3360M tonne
Total mineralized tonnage	1068M tonne
Mining capacity (per period)	290M tonne
Processing capacity (per period)	99M tonne
Total tailings capacity (per period)	81M m3
Reclamation sand (per period)	21M tonne
Reclamation OI (per period)	52M tonne

Table 3 - Compare of results for two directions

	Mining direction	
	West - East	East – West
# of all variables	9,115	7,915
# of integer variables	1,823	1,583
# of constraints	10,874	10,203
Solution gap (%)	5.2	5.9
Run time (sec)	2065	43
NPV (M$)	9,850	10,827

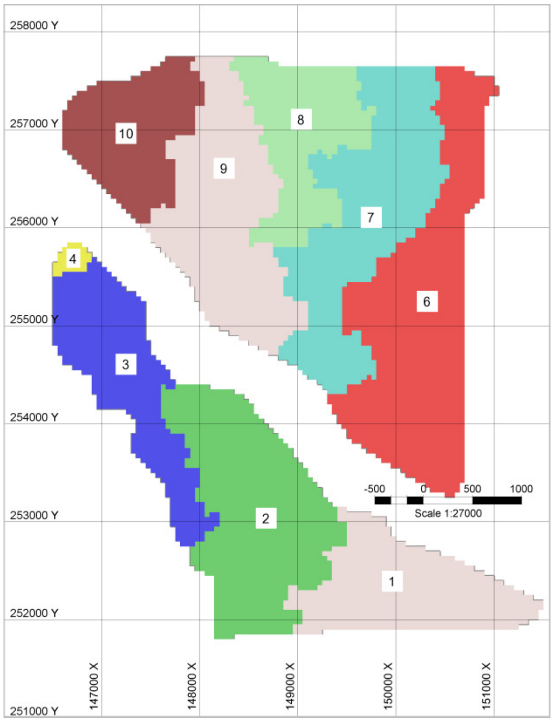

Figure 4- The mining schedule for East-West direction (bench 4)

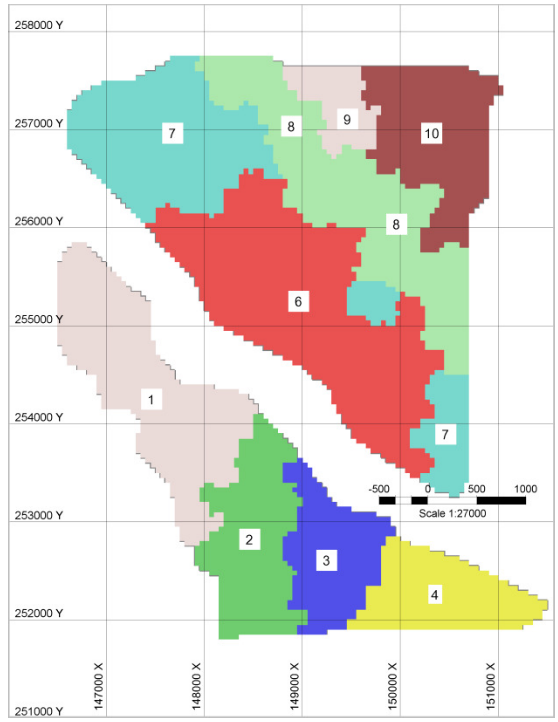

Figure 5- The mining schedule for East-West direction (bench 4)

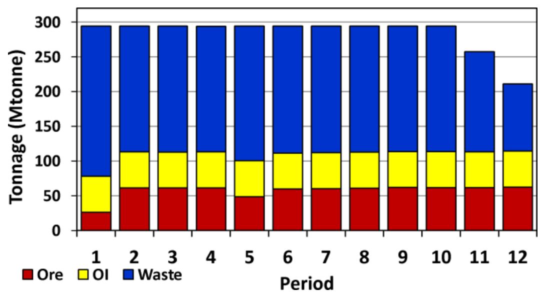

Figure 6- Ore, OI and Waste production plan for E-W direction

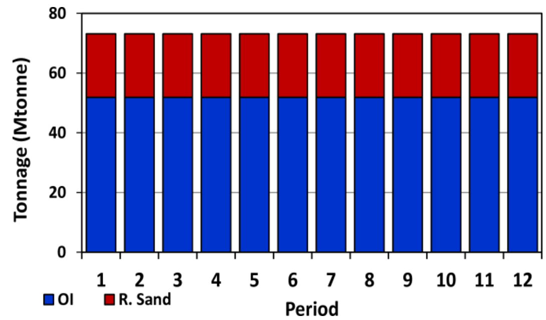

Figure 7- Reclamation material production plan for E-W direction

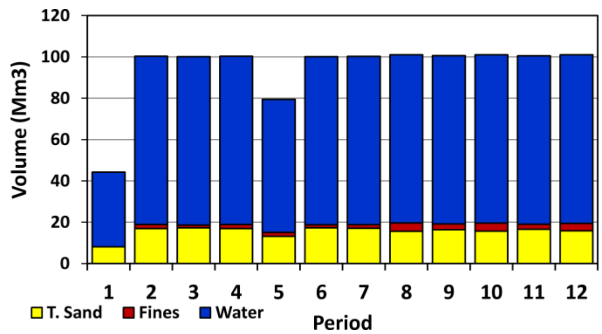

Figure 8- Tailings production plan for E-W direction

The optimal production plan for the east-west mining direction is presented in Figure 6. It includes the total tonnages for ore production, overburden and inter burden (OI) material and waste material that are extracted in each period. The graph shows that ore production is distributed in periods almost uniformly. It means that after period 2, the rate of ore feed to the processing plant is uniform. Moreover, except for the last two periods, the mining production rate is uniform too. That is because in the last periods, there is not enough material to follow uniform extraction pattern. Figure 7 shows the reclamation material production in 12 periods. OI denotes overburden and inter burden material, and R. Sand represents sand that is used for reclamation. The rate of production of reclamation material is completely uniform. That is because of the fact that production of reclamation material does not add any value to the objective function. In fact, it has a negative effect on the overall NPV, since there is only an extra cost associated with mining of reclamation material and sending that for capping. Therefore, the optimizer keeps the production of reclamation material to its minimum required level that is a uniform rate. Finally, Figure 8 illustrates the volume of total tailings slurry that is produced in each period. The total tailings volume is a summation of volumes of water, sand, and fine material.

CONCLUSIONS

Near-surface oil sands reserves are extracted through open-pit mining. Mine planning determines the optimal schedule for the extraction of the material in a way that maximizes the NPV. Extraction and processing of oil sands generates huge volume of waste, mostly in form of slurry known as tailings. One of the key factors that must be considered in mine planning is tailings management, because of two facts: (1) the available area for tailings impoundment is limited to the lease area and this limitation influences the mine planning indirectly, and (2) there are a number of environmental issues linked to the tailings ponds and therefore, oil sands operators must reclaim the mine site and tailings ponds before leaving the sites. These facts show that tailings management and reclamation planning must be integrated into the mine planning to have a comprehensive mine plan. In this paper, an integrated mine planning model is developed. It includes new sets of constraints for the capacity of tailings facility and the requirements for reclamation material. For the tailings management, the total volume of tailings slurry, plus the volumes of tailings components such as water, fine material and tailings coarse sand are included. For reclamation material, the integrated mine planning model is constrained by preparing the required tonnage of tailings coarse sand and over/inter burden material that will be used in later periods for capping in reclamation phase. Moreover, a new term is added to the objective function that minimizes the material handling cost of reclamation. An MILP model is developed for the integrated mine planning problem. The model is verified by testing the solutions on a real-case oil sands dataset. The results show that the optimal mine production schedule meets the tailings capacity constraints, as well as the reclamation material requirements in each period. Two horizontal directions (W-E and E-W) are compared to find the better mining direction for the case. As the next steps in this research, more solution methods, such as Lagrangian relaxation algorithm will be investigated to improve the solution time. Furthermore, in order to find the optimal solution for the large-scale problems in a reasonable time, the model will be modified for different resolutions for the mining unit, as mining-blocks, mining-cuts, and mining-panels.

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APPENDIX

The mixed integer linear programming model includes the following notation:

Sets

$K = \{1,...,K\}$: Set of all mining cuts in the model.

$J = \{1,...,J\}$: Set of all phases (push-backs) in the model.

$U = \{1,...,U\}$: Set of all material destinations in the model.

$C_k(L)$: For each mining-cut k, there is a set $C_k(L) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts above mining-cut k that must be extracted prior to extraction of mining-cut k, where L is the total number of mining-cuts in the set $C_k(L)$.

$M_k(P)$: For each mining-cut k, there is a set $M_k(P) \subset \boldsymbol{K}$ defining the immediate predecessor mining-cuts in a specified horizontal mining direction that must be extracted prior to extraction of mining-cut k at the specified level, where P is the total number of mining-cuts in the set $M_k(P)$.

$B_j(H)$: For each phase j, there is a set $B_j(H) \subset K$ defining the mining-cuts within the immediate predecessor pit phases (push-backs) that must be extracted prior to extracting phase j, where H is an integer number representing the total number of mining-cuts in the set $B_j(H)$.

Indices, Subscripts and Superscript

A parameter, f, can take indices, subscripts, and superscripts in the format $f_{k,j}^{u,e,t}$. Where:

$t \in \{1,......,T\}$: Index for periods.

$k \in \{1,......,K\}$: Index for mining-cuts.

$e \in \{1,......,E\}$: Index for elements of interest in each mining-cut.

$j \in \{1,......,J\}$: Index for phases (pushbacks).

$u \in \{1,......,U\}$: Index for material destinations.

D,S,M,P: Subscripts and superscripts for overburden and inter burden material, tailings sand, mining and processing respectively.

Parameters

$d_k^{u,t}$: Discounted profit obtained by extracting mining-cut k and sending it to destination u in period t.

$r_k^{u,t}$: Discounted revenue obtained by selling the final products within mining-cut k in period t if it is sent to destination u, minus the extra discounted cost of mining all the material in mining-cut k as ore and processing at destination u.

$n_k^{u,t}$: Extra discounted cost of mining the over/inter burden material of the mining-cut k in period t and sending it for reclamation in destination u.

$m_k^{u,t}$: Extra discounted cost of producing tailings sand from mining-cut k in period t and sending it for reclamation in destination u.

$q_k^{u,t}$: Discounted cost of mining all the material in mining-cut k in period t as waste and sending it to destination u.

g_k^e : Average grade of element e in the ore portion of mining-cut k.

$\underline{g}^{u,t,e}$: Lower bound on the required average head grade of element e in period t at processing destination u.

$\overline{g}^{u,t,e}$: Upper bound on the required average head grade of element e in period t at processing destination u.

f_k^o : Average percentage of fines in the ore portion of mining-cut k.

$\underline{f}^{u,t,o}$: Lower bound on the required average fines percentage of ore in period t at processing destination u.

$\overline{f}^{u,t,o}$: Upper bound on the required average fines percentage of ore in period t at processing destination u.

f_k^c : Average percentage of fines in the over/inter burden reclamation material portion of mining-cut k.

$\underline{f}^{u,t,c}$: Lower bound on the required average fines percentage of over/inter burden material in period t at reclamation destination u.

$\overline{f}^{u,t,c}$: Upper bound on the required average fines percentage of over/inter burden material in period t at reclamation destination u.

o_k : Ore tonnage in mining-cut k.

w_k : Waste tonnage in mining-cut k.

d_k : Over/inter burden material tonnage in mining-cut k.

l_k : Tailings sand material tonnage in mining-cut k.

t_k : Tailings tonnage produced downstream from extracting all of the ore from mining-cut k.

f_k : Fines tonnage produced downstream from extracting all of the ore from mining-cut k.

s_k : Sand tonnage produced downstream from extracting all of the ore from mining-cut k.

r_k : Water tonnage produced downstream from extracting all of the ore from mining-cut k.

T_{Mu}^{t} : Upper bound on mining capacity (tonnes) in period t.

T_{Ml}^{t} : Lower bound on mining capacity (tonnes) in period t.

$T_{Pu}^{u,t}$: Upper bound on processing capacity (tonnes) in period t at destination u.

$T_{Pl}^{u,t}$: Lower bound on processing capacity (tonnes) in period t at destination u.

$T_{Cu}^{u,t}$: Upper bound on over/inter burden reclamation material requirement (tonnes) in period t at destination u.

$T_{Cl}^{u,t}$: Lower bound on over/inter burden reclamation material requirement (tonnes) in period t at destination u.

$T_{Nu}^{u,t}$: Upper bound on tailings sand reclamation material requirement (tones) in period t at destination u.

$T_{Nl}^{u,t}$: Lower bound on tailings sand reclamation material requirement (tones) in period t at destination u.

$T_{Tu}^{u,t}$: Upper bound on capacity of tailings facility (tones) in period t at destination u.

$T_{Tl}^{u,t}$: Lower bound on capacity of tailings facility (tones) in period t at destination u.

$T_{Fu}^{u,t}$: Upper bound on capacity of fine material (tones) in period t at destination u.

$T_{Fl}^{u,t}$: Lower bound on capacity of fine material (tones) in period t at destination u.

$T_{Su}^{u,t}$: Upper bound on capacity of tailings sand (tones) in period t at destination u.

$T_{Sl}^{u,t}$: Lower bound on capacity of tailings sand (tones) in period t at destination u.

$T_{Wu}^{u,t}$: Upper bound on capacity of tailings water (tones) in period t at destination u.

$T_{Wl}^{u,t}$: Lower bound on capacity of tailings water (tones) in period t at destination u.

$r^{u,e}$: Proportion of element e recovered (processing recovery) if it is processed at destination u.

$p^{e,t}$: Price of element e in present value terms per unit of product.

$cs^{e,t}$: Selling cost of element e in present value terms per unit of product.

$cp^{u,e,t}$: Extra cost in present value terms per tonne of ore for mining and processing at destination u.

$cl^{u,t}$: Extra cost in present value terms for mining and shipping a tonne of over/inter burden material for reclamation at destination u.

$cu^{u,t}$: Extra cost in present value terms for mining and shipping a tonne of tailings sand material for reclamation at destination u.

cm^{t} : Cost in present value terms of mining a tonne of waste in period t.

Decision Variables

$x_k^{u,t} \in [0,1]$: A continuous variable representing the portion of ore from mining-cut k to be extracted and processed at destination u in period t.

$w_k^{u,t} \in [0,1]$: A continuous variable representing the portion of OI material from mining-cut k to be extracted and used for reclamation at destination u in period t.

$v_k^{u,t} \in [0,1]$: A continuous variable representing the portion of tailings sand material from mining-cut k to be extracted and used for reclamation at destination u in period t.

$y_k^{t} \in [0,1]$: A continuous variable representing the portion of mining-cut k to be mined in period t, which includes ore, over/inter burden material, tailings sand and waste.

$b_k^{t} \in [0,1]$: A binary integer variable controlling the precedence of extraction of mining-cuts. b_k^t is equal to one if the extraction of mining-cut k has started by or in period t, otherwise it is zero.

$c_j^{t} \in [0,1]$: A binary integer variable controlling the precedence of mining phases. c_j^t is equal to one if the extraction of phase j has started by or in period t, otherwise it is zero.

Modeling of Economic Mining-Cut Value

$$d_k^{u,t} = r_k^{u,t} - q_k^{u,t} - n_k^{u,t} - m_k^{u,t}$$

$$\forall t \in \{1,...,T\},\ u \in \{1,...,U\},\ k \in \{1,...,K\}$$

Where:

$$r_k^{u,t} = \sum_{e=1}^{E} o_k \times g_k^e \times r^{u,e} \times \left(p^{e,t} - cs^{e,t} \right) - \sum_{e=1}^{E} o_k \times cp^{u,e,t}$$

$$\forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,.., K\}$$

$$q_k^t = \left(o_k + d_k + w_k \right) \times cm^t \quad \forall t \in \{1,..,T\}, k \in \{1,.., K\}$$

$$n_k^{u,t} = d_k \times cl^{u,t} \quad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,.., K\}$$

$$m_k^{u,t} = l_k \times cu^{u,t} \quad \forall t \in \{1,..,T\}, u \in \{1,..,U\}, k \in \{1,.., K\}$$

The MILP Formulation

The objective functions of the MILP model for strategic and operational production plan for oil sands mining can be formulated as: i) maximizing the NPV and ii) minimizing the reclamation cost.

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(r_k^{u,t} \times x_k^{u,t} - q_k^t \times y_k^t \right) \right)$$

$$Min \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t} \right) \right)$$

These two functions can be combined as a single objective function, formulated as:

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \begin{bmatrix} \left(r_k^{u,t} \times x_k^{u,t} - q_k^t \times y_k^t \right) \\ -(n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t}) \end{bmatrix} \right)$$

The complete MILP model comprising of the combined objective function and constraints is formulated as:

Objective function:

$$Max \sum_{u=1}^{U} \sum_{t=1}^{T} \sum_{j=1}^{J} \left(\sum_{k \in B_j} \begin{bmatrix} \left(r_k^{u,t} \times x_k^{u,t} - q_k^t \times y_k^t \right) \\ -\left(n_k^{u,t} \times w_k^{u,t} + m_k^{u,t} \times v_k^{u,t}\right) \end{bmatrix} \right)$$

Subject to:

$$T_{Ml}^t \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(o_k + w_k + d_k \right) \times y_k^t \right) \le T_{Mu}^t$$

$$\forall t \in \{1,...,T\}$$

$$T_{Pl}^{u,t} \le \sum_{j=1}^{J} \left(\sum_{k \in B_j} \left(o_k \times x_k^{u,t} \right) \right) \le T_{Pu}^{u,t}$$

$$\forall t \in \{1,...,T\}, \ u \in \{1,...,U\}$$

$$T_{Cl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(d_k \times w_k^{u,t}\right)\right) \le T_{Cu}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$T_{Nl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(l_k \times v_k^{u,t}\right)\right) \le T_{Nu}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$\underline{g}^{u,t,e} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} g_k^e \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{g}^{u,t,e}$$

$$\forall t \in \{1,..,T\},\ u \in \{1,..,U\}, e \in \{1,..,E\}$$

$$\underline{f}^{u,t,o} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^o \times o_k \times x_k^{u,t} \Big/ \sum_{k\in B_j} o_k \times x_k^{u,t}\right) \le \overline{f}^{u,t,o}$$

$$\forall t \in \{1,..,T\}, u \in \{1,..,U\}$$

$$\underline{f}^{u,t,c} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j} f_k^c \times d_k \times w_k^{u,t} \Big/ \sum_{k\in B_j} d_k \times w_k^{u,t}\right) \le \overline{f}^{u,t,c}$$

$$\forall t \in \{1,..,T\}, u \in \{1,..,U\}$$

$$T_{Tl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(t_k \times x_k^{u,t}\right)\right) \le T_{Tu}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$T_{Fl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(f_k \times x_k^{u,t}\right)\right) \le T_{Fu}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$T_{Sl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(s_k \times x_k^{u,t}\right)\right) \le T_{Su}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$T_{Wl}^{u,t} \le \sum_{j=1}^{J}\left(\sum_{k\in B_j}\left(r_k \times x_k^{u,t}\right)\right) \le T_{Wu}^{u,t}$$

$$\forall t \in \{1,\ldots,T\},\ u \in \{1,\ldots,U\}$$

$$\sum_{u=1}^{U}\left(o_k \times x_k^{u,t} + d_k \times w_k^{u,t}\right) \le \left(o_k + d_k\right) \times y_k^t$$

$$\forall t \in \{1,..,T\},\ k \in \{1,..,K\}$$

$$\sum_{u=1}^{U}\left(l_k \times v_k^{u,t}\right) \le \sum_{u=1}^{U}\left(o_k \times x_k^{u,t}\right)$$

$$\forall t \in \{1,..,T\},\ k \in \{1,..,K\}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} x_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} w_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\}$$

$$\sum_{u=1}^{U}\sum_{t=1}^{T} v_k^{u,t} \le 1 \qquad \forall k \in \{1,..,K\}$$

$$b_k^t - \sum_{i=1}^{t} y_s^i \le 0$$

$$\forall t \in \{1,...,T\},\ k \in \{1,...,K\},\ s \in C_k(L)$$

$$b_k^t - \sum_{i=1}^{t} y_r^i \le 0$$

$$\forall t \in \{1,...,T\},\ k \in \{1,...,K\},\ r \in M_k(P)$$

$$\sum_{i=1}^{t} y_k^i - b_k^t \le 0 \qquad \forall t \in \{1,...,T\},\ k \in \{1,...,K\}$$

$$b_k^t - b_k^{t+1} \le 0 \qquad \forall t \in \{1,...,T-1\},\quad k \in \{1,...,K\}$$

$$H \times c_j^t - \sum_{i=1}^{t} y_h^i \le 0$$

$$\forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_j(H)$$

$$\sum_{i=1}^{t} y_h^i - H \times c_j^t \le 0$$

$$\forall t \in \{1,...,T\},\ j \in \{1,...,J\},\ h \in B_{j+1}(H)$$

$$c_j^t - c_j^{t+1} \le 0 \quad \forall t \in \{1,...,T-1\},\quad j \in \{1,...,J\}$$

$$\sum_{t=1}^{T} y_k^t = 1 \qquad \forall k \in \{1, \ldots, K\}$$