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Fifteen Years of Field Experience in LNG Expander Technology

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ABSTRACT

 From the earliest days of gas liquefaction, the thermodynamic process advantage of employing cryogenic liquefied gas expanders in place of throttling valves was very well recognized. The available technology was unable to offer reliable cryogenic liquefied gas expanders until the aerospace industry developed materials and designs suitable for the cryogenic environment.

LNG expanders reduce the high pressure of the condensed LNG by converting the static pressure energy of the fluid into electricity and sub-cooling the refrigerated LNG. The Carnot efficiency of the liquefaction process is significantly increased by using LNG expanders, resulting in a very short amortization time of less than six months for the financial investment in LNG expanders.

During the past fifteen years cryogenic LNG expanders have experienced an accelerated development in design and performance. Every newly built LNG liquefaction plant since 1996 has included two or more LNG expanders operating in each train.

The evolutionary design and performance of LNG expanders is presented, discussed, and fifteen years of practical field experience is demonstrated. Particular linear and non-linear phenomena in rotor-dynamics and fluid dynamics that are specific to the design and operation of single phase and twophase LNG expanders are described. An outlook for upcoming and future requirements of the LNG Expander Technology indicates trends towards both much higher and much smaller flow rates, together with much higher pressures. New design concepts to meet these future requirements are also disclosed.

INTRODUCTION

 The modern natural gas liquefaction process is based upon the first continuous process to liquefy air independently invented in 1895 by Carl von Linde in Germany and W. Hampson in the UK. In three steps the process compresses, cools and then expands the gas to a lower temperature. This triple step is then repeated until the gas condenses.

Figure 1 shows the schematic of the Linde-Hampson gas

liquefaction cycle for air. Piston compressors were used to compress the gas to a higher pressure, resulting in an increase in temperature. The compressed gas passes through a heat exchanger cooling the compressed gas and through a regenerator further cooling the compressed gas. The compressed and refrigerated gas is then expanded across a throttling valve, called a Joule-Thomson (J-T) valve, to reduce the pressure, cooling the gas to a still lower temperature.

The temperature of the gas at this stage is significant lower than the original gas temperature and the refrigerated gas is used as a cooling medium and passes through the regenerator to the piston compressor to repeat the cooling cycle. Each cycle further reduces the gas temperature until the liquefaction temperature is achieved and liquefied gas is condensing into the container.

Cycle for Air from 1895

The efficiency of the first continuous air liquefaction process was very low and the initial improvement was to increase the total pressure of the cooling cycle, including the pressure in the liquefied gas container which also functioned as a phase separator. At the outlet of this container is a flashing Joule-Thomson valve reducing the pressure of the condensed liquid, causing a partial vaporization of the liquid. This vaporization causes additional significant cooling of the remaining liquid due to evaporation cooling. The remaining vapour re-circulates through the regenerator to the compressor.

Further improvements of the Linde-Hampson liquefaction cycle were achieved by replaced the oscillating piston compressor with a more efficient rotating axial or radial gas compressor and the J-T valve with a rotating gas expander. The gas expander substantially increased the efficiency of the liquefaction process by extracting work from the compressed gas and reducing its enthalpy.

This process of liquefying has been applied to all gases and over the past fifty years large liquefaction plants for natural gas have developed using the same basic method.

INITIAL CONCEPT OF LNG EXPANDERS

 In December 1979, Henri Paradowski, (Paradowski 1979), proposed the use of "a cryogenic hydraulic turbine" to sub-cool the pressurized and condensed LNG to a low temperature in order to save energy in the natural gas liquefaction process. He received a US Patent in June 1982.

Figure 2: US Patent 4,334,902 Showing the Proposed Cryogenic Hydraulic Turbines

Figure 2 shows the drawing from US Patent 4,334,902 with the LNG expander (item 19) and the cryogenic propane cooling cycle expander (item 7) replacing the J-T valves (items 22 and 10).

The first cryogenic hydraulic turbine was built and installed in 1994 at an LNG and Helium liquefaction plant in Kansas. Shortly after in 1996, LNG expanders were installed at an LNG plant in Malaysia. This first generation LNG expander only partially replaced the flashing J-T valve with the outlet pressure approximately 500 kPa (72.5psia) above the saturation pressure to avoid vapour formation within the expander. The remaining pressure is then expanded across a flashing J-T valve.

Flashing J-T valves vaporize a small part of the LNG and the vaporization heat is extracted from the remaining liquid, thus sub-cooling the liquid by evaporation. A phase separator downstream the flashing J-T valve separates the vapour part from the liquid part.

Figure 3: LNG Expander at the LNG plant in Malaysia

Figure 3 shows the early design LNG expander installed in Malaysia. Coyle and Patel (2005) describe the design of this LNG expander as having three basic parts, the hydraulic turbine, the cryogenic shaft seal and the air cooled explosion proof induction generator. The hydraulic turbine has variable guide vanes to adjust to different flow rates with the cryogenic shaft seal providing separation between the turbine hydraulics at LNG temperature and the environmental temperature.

Figure 4 shows the LNG and Mixed Refrigerant (MR) expanders at the LNG liquefaction plant in Nigeria. MR expanders are those cryogenic propane cooling cycle expanders described in the Paradowski patent under item 19.

Figure 4: LNG and MR Expanders at an LNG Plant in Nigeria

At the Gastech 96 conference in Vienna, Austria, Kimmel (1996) proposed cryogenic hydraulic turbines with variable speed to adjust the performance to different flow rates. Figure 5 shows the cross sectional drawing of this variable speed LNG expander with geometrically fixed guide vanes.

The expander has radial inflow reaction turbine hydraulics which self limit maximum speed, with an induction generator mounted on an integral shaft. The entire unit, including the turbine and generator, is totally submerged in LNG. Japikse, D and Baines, N. C (1995) judge a radial inflow turbine runner as more effective than a mixed flow or axial runner in applications where a high work output is required. To expand high differential pressures, multi-stage turbines with multiple runners are utilized.

The LNG expander is equipped with a TEM®, a unique thrust equalizing mechanism, Weisser (1997), to balance the hydraulic thrust, significantly extending the lifetime of the LNG lubricated bearings. Thrust is eliminated in a particular and field proven manner. A small percentage of the inflow cryogenic liquid is required to cool the generator and to lubricate the ball bearings, and since this liquid stream contains pressure energy it can be used for compensating the axial thrust. For this purpose the TEM, a two orifice system, one fixed and one variable, is built into the turbine components. This system operates in a stable condition, offers excellent damping characteristics and has been proven to function correctly for LNG expander applications.

Figure 5: Cross Sectional Drawing of the Variable Speed LNG Expander

The cryogenic expander is entirely submerged in LNG and has no dynamic rotating shaft seals, no coupling between expander and generator, and operates safely within a stainless steel containment vessel designed and tested in accordance with ASME pressure vessel codes.

The electrical cable penetrations are static and fitted with dual and redundant in-line feed-through seals. The interspace is connected on site to a nitrogen barrier which provides an inert environment for the generator power cables.

Figure 6: Variable Speed Expander Characteristic

Variable speed turbines have been used for many years in hydropower units, where variable differential heads and flow rates are prevalent. Figure 6 shows a typical range of performance curves of these radial inflow turbines at different running speeds. The corresponding efficiency curves illustrate that the peak efficiency remains constant over a wide range of speeds. Figure 7 shows a variable speed LNG expander at the LNG test stand in Sparks, Nevada, before shipping to an LNG liquefaction plant in Oman.

Figure 7: Variable Speed LNG Expander on the Test Stand

Figure 8 illustrates a typical schematic for the installation of LNG expanders in a liquefaction plant. The pressurized LNG passes through the inlet isolation valve and through the LNG expander. The expander outlet pressure is much lower than the inlet pressure and the pressure energy is converted into electrical energy. The main flow control valve is located downstream of the expander to maintain the back pressure on the expander eliminating any possible vaporization caused by sudden pressure drops. Downstream of the control valve is the outlet isolation valve. During maintenance of the expander, both isolation valves are closed and the pressurized LNG is passed through the J-T valve for isenthalpic expansion. In case of an expander emergency shutdown the J-T valve opens as at least one of the isolation valves is closed.

Figure 8: Typical Installation Schematic for an LNG Expander

ADVANCED DESIGN OF LNG EXPANDERS

Conventional turbine technology developed from hydropower, where flow is directed from a higher elevation to a lower elevation, so that the high pressure inlet is located above the low pressure outlet. This configuration is natural when energy is to be extracted from waterfalls and river rapids. For the application of LNG expanders, the fluid power is given by the pressure of the LNG stream, which is practically independent on the elevation.

The first generation of LNG expanders followed the conventional hydropower design with both flow direction and pressure gradient downward, neglecting the fluid properties of the compressible LNG. When compressible fluids expand convective forces in an upward direction are created. This buoyancy enhances upward fluid flow but impedes downward flow.

In LNG expanders the axial thrust caused by the pressure gradient of the LNG is in the same direction as the LNG flow. In the case of downward flow the thrust force and rotor weight add together, whilst for upward flow the rotor weight is subtracted from the thrust force. The difference of the total net force is therefore two times the weight of the rotor. Since the axial net force has to be balanced by the thrust balancing device using a certain amount of pressurized LNG, this amount of LNG is significantly reduced for the upward flow and increases the overall efficiency. Cords (2010) compares the downward

Figure 9: Complete Assembly of an LNG Expander with Downward Flow Design

and upward flow design for LNG expanders. Figure 9 shows the complete assembly of the conventional downward flow design and Figure 10 the advanced upward flow design.

The upward flow design has a lower centre of gravity than the downward flow design due to the greater weight of the induction generator, with its laminations and copper, compared to the hydraulic part manufactured from cryogenic aluminum. A low centre of gravity reduces vibration and offers additional mounting stability.

The advanced design for expanders with upward flow and upward pressure gradient also enables without modifications, expansion of saturated two-phase LNG with up to 10% volumetric vapour content. With some additional components the upward flow expander can expand saturated LNG to higher vapour content, even to 100% vapour. These design modifications are included in two-phase LNG expanders.

Figure 10: Complete Assembly of an LNG Expander with Upward Flow Design

This design of LNG expanders are straightforward to maintain. Having fixed geometry, the only moving part is the complete rotor with its induction generator, all mounted on a single shaft. The maximum plant differential pressure determines the maximum rotational, self limiting speed along the zero torque line. Thus there is no 'run-away' condition and the design speed is at a margin above this maximum speed.

The unique TEM balances the total hydraulic thrust, providing significantly extended bearing life. There are no regular daily or monthly maintenance tasks to undertake, there being no rotating seals or seal system, no oil lubricated bearings or lubeoil system and no actuators or similar ancillaries. Generator output, inlet and outlet conditions and vibration are monitored, providing a complete picture of the machine's condition trend in order to plan maintenance.

Compared to other designs, routine maintenance is achieved with a degree of ease due to the small size and weight of the components and relatively few ancillary components to maintain or remove before removal of the expander. The simplicity of the expander design, including ease of setting the TEM, enables assembly and disassembly with a minimum of special tools.

Hybrid bearings with steel inner and outer races, silicon nitride balls and molded resin separators have been successfully used to increase bearing life. Their greater resistance to debris and reduced wear in low lubricity liquids shows a significant extension of life.

Hydrostatic bearing have been utilizing by the manufacturer and could also be adopted with 'touchdown' ball bearings for start‐up and shutdown. Rotor dynamic studies of hydrostatic bearings at the upper, lower and tail bearing locations show improved separation and damping stability with the entire operating region of the expander shifted to a more conservative area. Endurance testing verifies hybrid touchdown bearings will endure axial forces during start‐up and shutdown with high axial force applied over several hundred start and stop cycles.

TWO-PHASE LNG EXPANDERS

In modern refrigeration process plants the liquefaction of natural gas is achieved at increasingly higher pressure and mass flow to improve the overall process efficiency. Trends for future LNG plants are naturally towards larger mass flow and higher pressure to secure the efficiency gains offered. The condensed and highly pressurized LNG is expanded across a liquid-vapour two-phase expander to a lower pressure suitable for storage and transportation of the liquefied gas. This expansion process generates a certain amount of vapour and the remaining liquid is further cooled. The objective of the twophase LNG expander is to increase the amount of liquid and to decrease the amount of vapour at the expander outlet.

Figure 11 compares the arrangement of conventional single phase LNG expanders with two-phase LNG expanders.

Figure 11: Conventional Single-Phase Versus Two-Phase LNG Expander

For the conventional process the outlet pressure of the LNG expander is approximately 500 kPa (72.5psia) above the saturation pressure and the remaining pressure energy is throttled across a flashing two-phase J-T valve. For the new process employing two-phase LNG expanders the outlet pressure of the expander is equal to the outlet pressure of the J-T valve in the conventional process. The two-phase LNG expander converts the total available pressure into electrical energy and reduces the enthalpy of the LNG, decreasing the amount of vapour and increasing the amount of liquid.

EULER TURBINE EQUATION APPLIED IN TWO-PHASE EXPANDERS

Leonhard Euler published his universal turbine equation, which applies to any type of turbine, under the title "A more complete theory of machines which are activated by their reaction to water" at the Academy of Science, Berlin in 1756. (Euler 1756)

In LNG services, the radial inflow turbine, a perfect embodiment of the Euler turbine theory, has become the preferred design for cryogenic liquid expanders. The Euler turbine Equation (1) states that the generated torque T of rotating turbine runners is equal to the difference of the angular momentum L at the inlet L1 and outlet L2.

$$
T = L1 - L2 \tag{1}
$$

The angular momentum L is equal to the product of the tangential velocity c of the fluid, the radial distance r to the centre of rotation and the mass flow \dot{m} per time. Equation (2)

$$
L = c r \dot{m} \tag{2}
$$

The angular momentum L is defined as positive if rotating in the same rotational direction with the runner, and negative if rotating in opposite direction. If the inlet and outlet angular momentum L1 and L2 are both positive then the maximum generated torque T_{max} is equal to the inlet angular momentum L1 if the outlet momentum L2 is equal to zero. Equation (3)

$$
T_{\text{max}} = L1
$$
 for $L2 = 0$ (3)

The efficiency of the turbine reaches a maximum value if the outlet angular momentum is equal to zero, because the fluid exits the turbine runner with no rotational energy. If the outlet angular momentum has a remaining rotational energy that is not recovered then the overall turbine efficiency is reduced. This reduction in overall efficiency can be eliminated by converting the remaining outlet rotational energy into static pressure energy by using a diffuser type flow straightener.

The generation of an optimal torque T_{opt} , larger than T_{max} is feasible if the outlet angular momentum L2 is negative giving a greater differential angular momentum between inlet angular momentum L1 and outlet momentum L2. Equation (4)

$$
T_{opt} = L1 - (-L2) = L1 + L2 > T_{max} \quad (4)
$$

The overall turbine efficiency is increased if the remaining outlet rotational energy is converted in to static pressure energy described above.

Figure 12 shows the general torque T as the difference of the inlet and outlet angular momentum L1 and L2. Torque T corresponds to the size of the hatched area, since the angular momentum is equal to the product of tangential velocity c and radial distance r. The mass flow m is a constant scaling value across the expander.

Fig 12: General Torque T

Figure 13 shows the maximum torque T_{max} with the outlet angular momentum L2 equal to zero corresponding to the hatched area.

Fig 13: Maximum Torque Tmax

Figure 14 shows the optimal torque T_{opt} with positive inlet angular velocity generating a negative outlet angular momentum increasing the differential angular momentum.

Fig 14: Optimal Torque Topt

The optimal torque is greater than the maximum torque. The fundamental Euler turbine equation with its possible variations of negative, positive and zero outlet angular momentum is applied in the design of the liquid-vapour two-phase LNG expander presented here.

DESIGN OF LIQUID-VAPOR TWO-PHASE LNG EXPANDERS

Essentially, the design of two-phase LNG expanders is very similar to existing turbine and expander technology. The hydraulic energy of the pressurized fluid is first transformed into kinetic energy and then into mechanical shaft power, which is converted to electric energy by an electrical induction generator.

Figure 15 shows the two-phase LNG expander assembly with vertical rotational axis, mounting on the head plate inside the pressure vessel. The pressurized LNG enters through a horizontal inlet nozzle at the lower vessel part and flows upwards between vessel and expander, enters the expander at the red colored nozzle ring and exits the expander and the vessel at the top through the vertical outlet nozzle. The main purpose of the pressure vessel is to protect the expander against mechanical damage and fire since LNG is a flammable liquid and aluminum is a flammable metal

The expander itself consists of three main parts, the housing, the hydraulic assembly and the electric induction generator. The hydraulic assembly is shown in the colors red, yellow, green and silver blue. The runner shown in color yellow and exducer shown in color green are mounted together on one rotating shaft with the generator shown in color brown.

Figure 15: Cross Section of the Two-Phase LNG Expander

Figure 16 shows the hydraulic assembly transforming pressure energy into shaft torque. The static, non-rotating, red colored nozzle ring is mounted on the expander housing. The nozzle ring converts the pressure energy of the fluid into kinetic energy generating the essential inlet angular momentum described by the Euler turbine equation. This energy transformation is achieved with high efficiency due to the specific design of the nozzle ring. The cross section of the fluid passage between the nozzle vanes is reduced in both vertical and radial direction. This three dimensional nozzle ring design efficiently produces an angular Euler momentum with minimal losses, and is most influential and fundamental for the turbine and expander performance.

The rotating radial inflow turbine runner of the expander consists of the grey colored shroud and the yellow colored hub with guide vanes in Figure 16. The fluid passes through the fluid passages formed by the inlet guide vanes, red with silver, entering the runner in angular and radial direction, exiting the runner in axial and vertical direction. The design of the runner as a radial inflow turbine is such that the outlet angular momentum is zero with no remaining rotational fluid energy at the exit of the runner and the entrance of the jet exducer.

Mounted on top of and rotating with the runner is the jet exducer. The exducer consisting of the green colored hub with guide vanes and the grey colored shroud. The jet exducer is a radial outflow turbine, also known as Hero's turbine or Euler turbine, with no inlet angular momentum. The purpose of the exducer is to generate a negative outlet angular momentum. As described above, a negative outlet angular momentum increases the differential angular Euler momentum resulting in an increased shaft torque and shaft power.

The guide vanes for the fluid flow through the exducer are helically wrapped around the shaft with cross sections increasing in size. The saturated LNG begins to vaporize at the inlet of the exducer forming a liquid-vapour two-phase fluid flow. As the volume of the vaporizing LNG increases and passes through the helical guide vanes the velocity of the fluid is increased. The increase in fluid velocity causes a drop in the pressure due to the Conservation of Energy Law. This drop in pressure increases the LNG vaporization at any location inside the guide vanes due to the thermodynamic properties.

Increased vaporization again increases the volume and fluid velocity, causing further pressure drop in the fluid. The described vaporization-volume-velocity-pressure cycle is effective at any location within the helical guide vanes and repeats itself until the liquid-vapour mixture exits the exducer with a high fluid velocity.

The high rotational exit velocity generates a large negative outlet momentum increasing the differential angular Euler momentum. The design of the jet exducer is applicable to liquid-vapour two-phase fluids of any ratio between vapour and liquid, including single-phase fluids as liquid or as vapour. Operating the LNG expander with variable rotational speed enables the expansion of single and two-phase fluids with smooth and uninterrupted transition across all phase ratios.

The large negative outlet angular momentum increases the shaft torque but also causes the fluid to exit the jet exducer with a remaining rotational kinetic energy. To recover this rotational kinetic energy by converting it into static pressure energy, the fluid passes through a so called condensation cone, also shown in Figure 16, designed like a turbine draft tube with guide vanes. The non rotating silver blue colored condensation cone is mounted on the expander housing.

The guide vanes are helically shaped with changing pitch, the pitch at the inlet being small and increasing continuously in an upward direction until the vanes are parallel to the shaft axis. The helical vanes are attached to the conically shaped centre piece. The condensation cone reduces the fluid velocity to the necessary axial fluid velocity at the outlet nozzle of the pressure vessel. Thus the rotational fluid energy is converted into pressure energy as a result of the Conservation of Energy Law transforming kinetic energy into static energy. The twophase LNG expander design presented performs the thermodynamic expansion of liquefied gases with optimal

Figure 16: Hydraulic Assembly for Two-Phase Expansion

process efficiency, complying in an advanced form with the fundamental Euler turbine equation. The theory of thermo-fluid dynamics describes the two-phase expansion process in detail across the LNG expander.

THERMO-FLUID DYNAMICS OF TWO-PHASE LNG EXPANDERS

 The power generated by turbine expanders can be calculated by applying mass, momentum and energy conservation laws. In the case of turbines driven by ideal incompressible non-viscous liquids, the generated theoretical maximum power P is equal to the product of volumetric flow Q and pressure difference Δp between turbine inlet and outlet expressed by Equation (5)

$$
P = Q \Delta p \qquad (5)
$$

In case of compressible non-viscous fluids or vapour gases the volumetric flow changes at any point across the turbine. The generated power has to be expressed in a differential term dP for a small differential pressure dp. Equation (6) expresses the volumetric flow Q change with the extracted generated power P and the pressure p.

$$
dP = Q[P,p] dp \quad (6)
$$

This equation for the mechanical expansion power can be transformed in terms of specific power or specific enthalpy h, specific volume v and pressure p. The thermo-physical properties state that the specific volume v is a function of two independent property variables, such as enthalpy h and pressure p, symbolized in Equation (7)

$$
v = v[h, p] \qquad (7)
$$

The theoretical maximum differential power generated by the expansion of two-phase fluids is described by the following non-linear differential Equation (8)

$$
dh = v [h,p] dp \quad (8)
$$

The specific volume v is a two variable function of the specific enthalpy h and the pressure p, and listed in thermo-physical properties tables. It is also expressed as a two variable polynomial function and the non-linear differential equation can be integrated, providing the total theoretical maximum power generated by expansion of two-phase fluids. The practical power is calculated from the theoretical power multiplied by the expander efficiency.

TWO-PHASE LNG EXPANDERS IN THE FIELD

 Two-phase expanders are applicable for single-phase liquid and for two-phase liquid-vapour LNG with stable and undisturbed transition between single and two-phase operation. Dependent on the particular liquefaction plant, the two-phase expander is installed for saturated LNG only or installed in combination with an expander for single-phase liquid and a two-phase expander.

Figure 17 shows the installation of the very first two-phase LNG expander in Poland in 2003. Figure 18 shows the installation during 2009 in Poland of the two-phase LNG tandem configuration expander and Figure 19 displays the upper and lower tandem expanders at the manufacturing plant.

Figure 20 shows the combination of a single-phase and a twophase LNG expander in tandem configuration. The lower expander reduces the high pressurized liquid LNG to the saturation pressure and the upper expander expands into saturated two-phase LNG conditions. The fluid flow direction is persistent in upward direction and parallel to the pressure gradient. The lower and the upper expander are able to operate at different rotational speeds to adjust to different fluid densities.

Figure 17: Installation of the Very First Two-Phase LNG Expander in Poland 2003

Figure 18: Installation of the Tandem Configuration in Poland in 2009

Figure 19: Two-Phase Expanders at the Manufacturing Plant

CONCLUSION

The current trends in production of LNG are towards larger plants with improved efficiency, operating at higher pressure and creating minimum environmental impact. These trends appear certain to continue into the future. For smaller reserves small liquefaction plants with high efficiency are also required.

For many years each new liquefaction plant has incorporated new technologies in combination with field proven technologies with the aim of improvements in output and efficiency. This trend will continue and needs plant owners, licensors, contractors and equipment manufacturers to continue cooperating with each others to reach this goal.

Figure 20: Combined Single and Two-Phase LNG Expanders in Tandem Configuration

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