A NEW COMPUTER PROGRAM FOR PUMP ROTORDYNAMIC ANALYSIS

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ABSTRACT

A new computer program for pump rotor analysis has been developed to improve the accuracy of critical speed, stability, and response calculations for multistage centrifugal pumps. The program capabilities include: (1) two nonlinear journal bearings, each with eight speed-dependent stiffness and damping coefficients and with flexible bearing supports, and, (2) up to seventeen seal stations with radial and angular seal misalignments and with each seal or impeller represented by full 4 × 4 speed-dependent stiffness, damping and inertia matrices. Also, speed-dependent steady-state impeller fluid forces are accounted for. The program has the additional capability to automatically account for the nonlinear variation of the bearing coefficients resulting from part of the rotor weight being supported by the seals.

This is believed to be the most advanced computer program currently available for pump rotor analysis. At the same time, the implementation is user friendly and can be run with different levels of sophistication, depending on user needs and available input data.

The program has been used to analyze a multistage centrifugal pump for which a natural frequency had been measured during normal operation. Both the addition of inertia coefficients to the seal model and the extension of the seal model to include moment coefficients had significant effects on rotordynamics. The natural frequency prediction, for example, improved significantly. The improved bearing coefficients, however, had little influence on the particular pump rotor analyzed.

INTRODUCTION

The continuing efforts to improve the performance of multistage centrifugal pumps have necessitated locating the operating speed of the pumps above the first bending critical speed of the pump rotor. As a result, multistage pumps are now so sensitive to instabilities and unbalances that comprehensive rotordynamic analyses have become mandatory in order to ensure pump reliability. Such rotordynamic analyses require specialized computer programs which can properly account for the rotor/stator interactions at both the seals, the impellers and the journal bearings. The history of the development of pump rotor programs is relatively short, starting in 1980 with Takagi, et al. [1], who incorporated the effect of the static load carrying capability of the seals on the dynamic characteristics of the bearings. In 1981-82, Gopalakrishnan, et al. [2], [3], included the stiffness effects of the seals into the rotor analysis in terms of the so-called Lomakin mass [4]. Subsequently, Bolleter, et al. [5], extended the pump rotor model to included 2 × 2 matrices of seal and impeller stiffness, damping and inertia coefficients as well as 2 × 2 matrices of bearing stiffness and damping coefficients. The notation "2 × 2" refers to a model which describes the force components (F_x, F_y) as a linear function of the displacement (x, y) and its derivatives. In 1987, a program with similar capabilities was described by Diewald and Nordmann [6].

The 2 × 2 seal coefficient matrices may be adequate for most seals but long seals, such as balance pistons, are likely to have large moment reactions in addition to the force reactions and therefore require 4 × 4 stiffness, damping and inertia coefficient matrices. A "4 × 4" model defines the force and moment components as a linear function of the displacement and rotation (pitch and yaw). Such matrices have been available for several years based on computer calculations [7]. The pump rotor program described in this paper has been developed to include these 4 × 4 seal coefficient matrices in the rotor analysis.

A second important problem, first addressed by Nikolajsen [8] and then by Takagi et al. [1], is the determination of the bearing stiffness and damping coefficients. These coefficients depend strongly on the static bearing eccentricities, which are unknown, because the seals carry most of the rotor weight. Current practice of assuming that the bearings carry all the rotor weight can lead to significant errors in the dynamic analysis, particularly if the pump rotor is short and stiff. The program described herein eliminates this problem by calculating the static bearing eccentricities for the statically indeterminate rotor before proceeding with the dynamic analysis.

The program can also account for both radial and angular seal misalignments, thus allowing the user to perform parametric studies of the effect of such misalignments on rotordynamics. Finally, the flexibility of the bearing pedestals can be included in the analysis.

The computer code was developed in FORTRAN on a VAX computer in the Turbomachinery Laboratory at Texas A&M University. The code is specifically adapted for analysis of multistage centrifugal pumps with two bearings and multiple seals and impellers along the span.

PROGRAM DEVELOPMENT

Static Analysis Method

The objective of the static analysis is to calculate the static bearing eccentricities, e, and the direction, α, of the static bearing loads, F_B, for the statically indeterminate pump rotor supported by two bearings and multiple seals (Figure 1). The bearing eccentricities are needed to determine the dynamic stiffness and damping coefficients of the bearings which are customarily defined as functions of the eccentricity ratios, (i.e., Lund [9]). The bearing coefficients found in the literature are usually given in a coordinate system with the x-axis along the direction of the static bearing load shown in Figure 1. The static load directions, α, for the pump bearings must therefore also be determined, so the coefficients can be transformed to the pump rotor coordinate system (XY) with horizontal and vertical axes as used in the subsequent rotordynamics analysis.

Static equilibrium of the rotor finite element model can be expressed by the following matrix equation of order four times the number of discrete shaft stations:

$$K_o\overline{X}=\overline{F}_o-\overline{F}_B-\overline{F}_S \quad (1)$$

where

K_o = unsupported shaft stiffness matrix

$\overline{X}$ = vector of radial and angular shaft displacements

$\overline{F}_o$ = vector of known external forces (e.g. rotor weight and static impeller forces

$\overline{F}_B$ = bearing forces (known nonlinear functions of bearing eccentricity given in the literature in nondimensional graphical or tabular form as a function of the Sommerfeld Number, see for example Ref. [9])

$\overline{F}_S$ = seal forces (known linear functions of seal eccentricities and nonlinear functions of rotational speed Ω)

Eq. (1) can be expanded to the following form to account for the radial and angular seal misalignments relative to the bearing centerline (Figure 2, and to account for the bearing backup structure stiffness, see Figure 1):

$$K\Delta\overline{X}=\overline{F}-(1+K_1)\overline{F}_B \quad (2)$$

where

$K=K_o$ with the seal stiffnesses added at the appropriate locations by the direct stiffness method.

$\Delta\overline{X}=\overline{X}$ except at the two bearing locations where $\Delta\overline{X}$ designates the shaft displacements relative to the flexible bearing backup structure (i.e. the X and Y components of the bearing eccentricities e)

$\overline{F}=\overline{F}_o$ with the additional forcing due to the seal misalignments included

I = identity matrix

$K_1=K$ with the columns associated with the bearing locations divided by the bearing backup structure stiffness κ

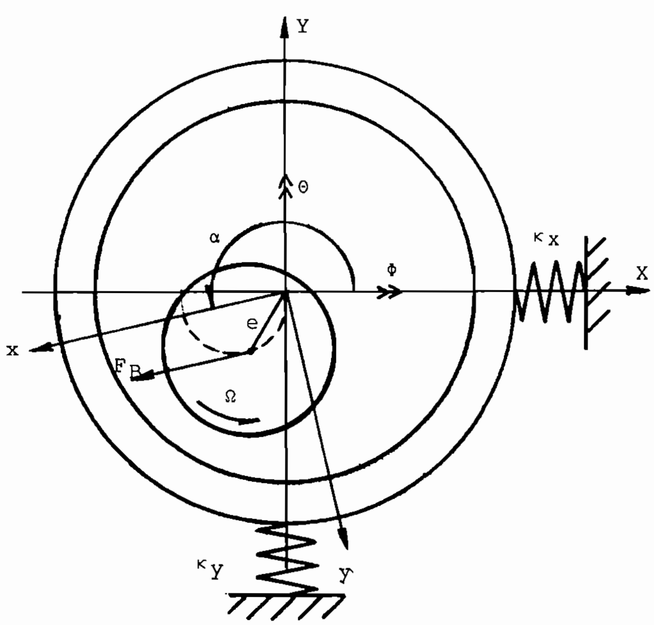

Figure 1. Bearing with Flexible Support.

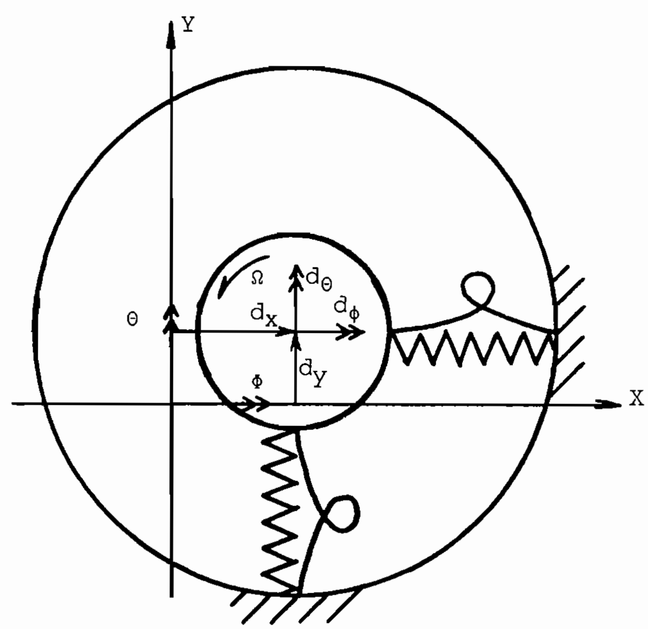

Figure 2. Seal with Misalignment.

For a complete derivation of Equation (2), please refer to Gajan [10].

Equation (2) is a nonlinear equation system from which the bearing eccentricities e and the directions of F_B can be found by Newton-Raphson iteration. Advantage is taken of the fact that some of the partial derivatives of the Jacobian matrix turn out to be the bearing stiffness coefficients which are known functions of the eccentricities.

An automatic starting procedure was developed for the iteration process which ensures consistent rapid convergence without the need for informed guesses on the part of the user. Further details are available in Gajan [10].

Dynamic Analysis Method

The objective of the dynamic analysis is to determine the damped critical speeds and the mode shapes of the pump rotor and also to determine the steady-state unbalance response. An existing finite element program called ARDS was selected and modified for this purpose. ARDS was developed by Nelson, et al. [11], and requires access to the IMSL subroutine library. ARDS sets up the equations of motion in complex matrix form and solves the generalized eigenvalue problem $(Ms^2 + Bs + K)x = 0$ by means of the QR-algorithm to find the damped natural frequencies and mode shapes. The unbalance response is found by solving the linear equations system $(K - M\Omega^2 + i\Omega b)\,x = F$.

The natural frequency and the steady-state response algorithms were extracted from ARDS and extended to accept speed dependent radial stiffness and damping coefficients for the bearings (2 × 2 coefficient matrices) and speed dependent radial and angular stiffness, damping and inertia coefficients for the seals (4 × 4 coefficients matrices). The bearing coefficients must be supplied by the user in table form as functions of the eccentricity ratio. The seal coefficients must also be available for at least three speeds distributed over the speed range. A curve fitting routine in the program then calculates them at any other speed.

The static analysis capability, described in the previous section, was added to automatically calculate the bearing stiffness and damping coefficients for the statically indeterminate rotor at each speed.

The program was tested extensively by comparison of results both with closed form solutions for simple rotor systems and with numerical results from other available computer programs with related capabilities [8].

SAMPLE ANALYSIS

An eleven stage centrifugal pump, made by Dresser Pacific [12], was analyzed to determine what effect the new program capabilities have on the predicted rotordynamic behavior. The pump rotor is shown in Figure 3. It is 97 inches long and has an operating speed of 6,600 rpm.

Case 1

The rotor was first analyzed with the standard program capabilities generally available in the pump industry. They include speed dependent radial stiffness and damping coefficients at both the seals and the bearings (2 × 2 coefficient matrices) with the bearing coefficients calculated with the assumption that each bearing carries one half of the total weight of the rotor.

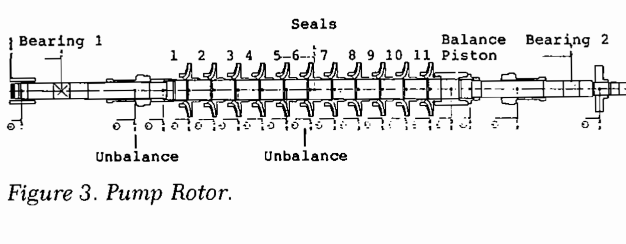

Figure 3. Pump Rotor.

The seal forces are thus expressed as

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix}_S = \begin{bmatrix} K & k \\ -k & K \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} C & c \\ -c & C \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$$

and the bearing forces are

$$\begin{bmatrix} F_x \\ F_y \end{bmatrix}_B = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix}$$

Case 2

The rotor was then re-analyzed with the seal model extended to include speed dependent radial and angular stiffness and damping coefficients. i.e.,

$$\begin{Bmatrix} F_x \\ F_y \\ M_y \\ M_x \end{Bmatrix}_S = \begin{bmatrix} K & k & K_{\varepsilon\alpha} & -k_{\varepsilon\alpha} \\ -k & K & -k_{\varepsilon\alpha} & -K_{\varepsilon\alpha} \\ K_{\alpha\varepsilon} & k_{\alpha\varepsilon} & K_\alpha & -k_\alpha \\ k_{\alpha\varepsilon} & -K_{\alpha\varepsilon} & k_\alpha & K_\alpha \end{bmatrix} \begin{Bmatrix} X \\ Y \\ \Theta \\ \Phi \end{Bmatrix} + \begin{bmatrix} C & c & C_{\varepsilon\alpha} & -c_{\varepsilon\alpha} \\ -c & C & -c_{\varepsilon\alpha} & -C_{\varepsilon\alpha} \\ C_{\alpha\varepsilon} & c_{\alpha\varepsilon} & C_\alpha & -c_\alpha \\ c_{\alpha\varepsilon} & -C_{\alpha\varepsilon} & c_\alpha & C_\alpha \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Theta} \\ \dot{\Phi} \end{Bmatrix}$$

Case 3

Inertia coefficients were then added to the seal model giving

$$\begin{Bmatrix} F_x \\ F_y \\ M_y \\ M_x \end{Bmatrix} = \begin{bmatrix} K & k & K_{\varepsilon\alpha} & -k_{\varepsilon\alpha} \\ -k & K & -k_{\varepsilon\alpha} & -K_{\varepsilon\alpha} \\ K_{\alpha\varepsilon} & k_{\alpha\varepsilon} & K_\alpha & -k_\alpha \\ k_{\alpha\varepsilon} & -K_{\alpha\varepsilon} & k_\alpha & K_\alpha \end{bmatrix} \begin{Bmatrix} X \\ Y \\ \Theta \\ \Phi \end{Bmatrix} + \begin{bmatrix} C & c & C_{\varepsilon\alpha} & -c_{\varepsilon\alpha} \\ -c & C & -c_{\varepsilon\alpha} & -C_{\varepsilon\alpha} \\ C_{\alpha\varepsilon} & c_{\alpha\varepsilon} & C_\alpha & -c_\alpha \\ c_{\alpha\varepsilon} & -C_{\alpha\varepsilon} & c_\alpha & C_\alpha \end{bmatrix} \begin{Bmatrix} \dot{X} \\ \dot{Y} \\ \dot{\Theta} \\ \dot{\Phi} \end{Bmatrix} + \begin{bmatrix} M & 0 & M_{\varepsilon\alpha} & 0 \\ 0 & M & 0 & -M_{\varepsilon\alpha} \\ M_{\infty\varepsilon} & 0 & M_\alpha & 0 \\ 0 & -M_{\alpha\varepsilon} & 0 & M_\alpha \end{bmatrix} \begin{Bmatrix} \ddot{X} \\ \ddot{Y} \\ \ddot{\Theta} \\ \ddot{\Phi} \end{Bmatrix}$$

Case 4

Finally, the static analysis capability, described in the previous section, was also activated to improve the bearing coefficients.

The corresponding Campbell diagrams are shown in Figures 4, 5, 6 and 7. The damped natural frequencies have been labeled, F, for forward whirl and, B, for backward whirl. The logarithmic decrements are also shown at regular intervals along

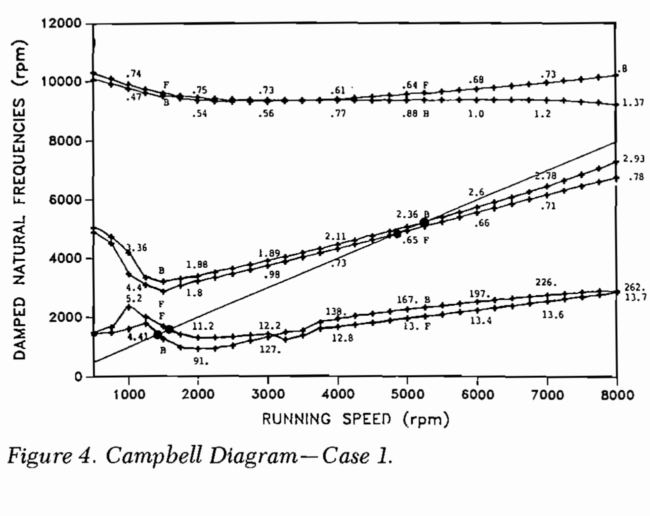

Figure 4. Campbell Diagram—Case 1.

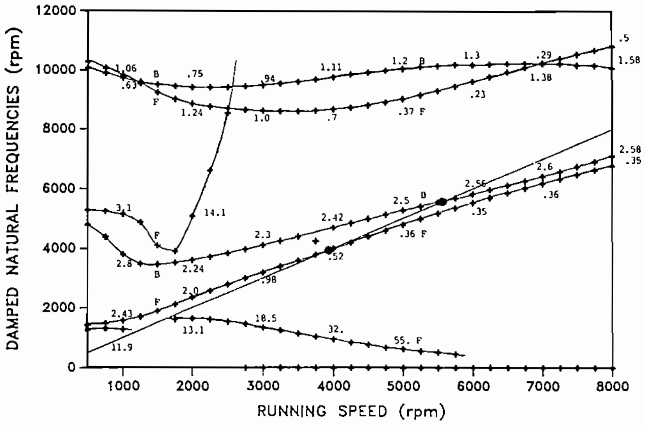

Figure 5. Campbell Diagram—Case 2.

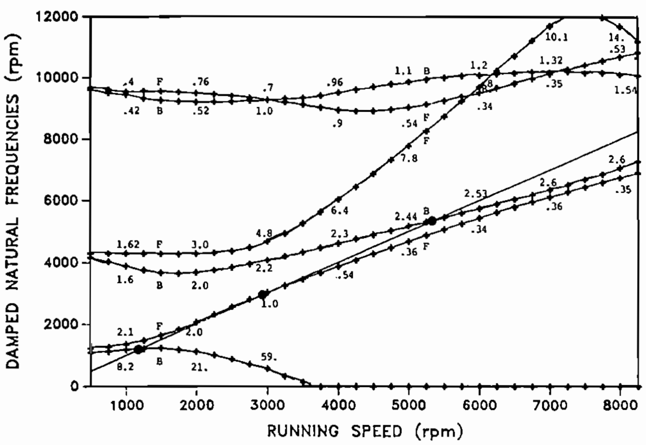

Figure 6. Campbell Diagram—Case 3.

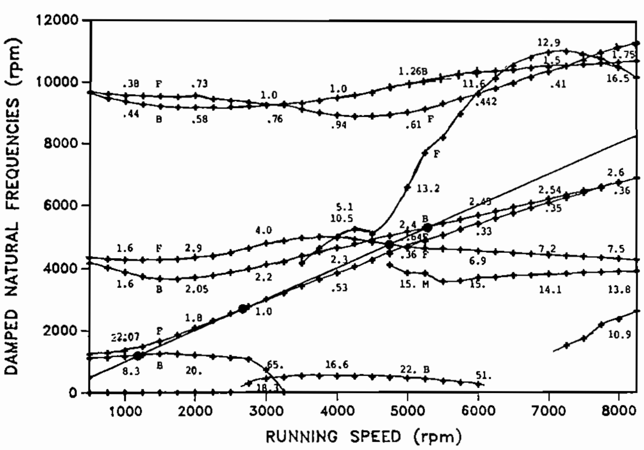

Figure 7. Campbell Diagram—Case 4.

the frequency lines. The straight line with a slope of, 1, is the run line corresponding to synchronous vibrations. The intersections between the run line and the frequency lines designate potential critical speeds (marked by black circles).

The differences between the four Campbell diagrams are quite pronounced. Case 1 looks relatively ordinary, with each of the three natural frequencies split into two which stay close together throughout the speed range. The only critical speed which is expected to show up is at about 5000 rpm. Cases 2 and 3 look more unusual with two of the three natural frequencies moving apart and the first forward natural frequency remaining close to the run line. If a critical speed exists, it will be associated with the first forward natural frequency. In Case 2, the log dec is only 0.52 where this natural frequency crosses the run line, so a critical speed is expected. In Case 3, the natural frequency is well damped with a log dec of one as it crosses the run line at 3000 rpm. It then loses damping as it slowly moves away from the run line. Thus, the bandwidth determines whether a critical speed exists. Case 4 looks very unusual with several additional natural frequencies appearing throughout the speed range. This happens when the corresponding bending modes are initially overdamped and then eventually become underdamped at higher speeds. Here again, it is difficult to predict whether a critical speed exists.

The damped mode shapes for the nearly synchronous forward whirl frequencies at 1500 rpm and at 5000 rpm are shown in Figures 8 and 9.

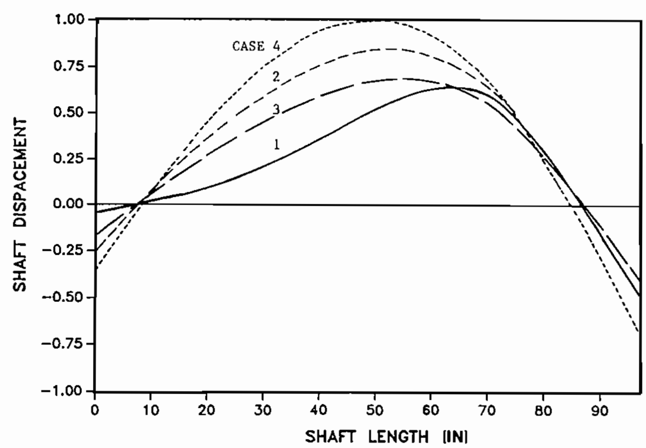

Figure 8. Mode Shapes—1500 RPM.

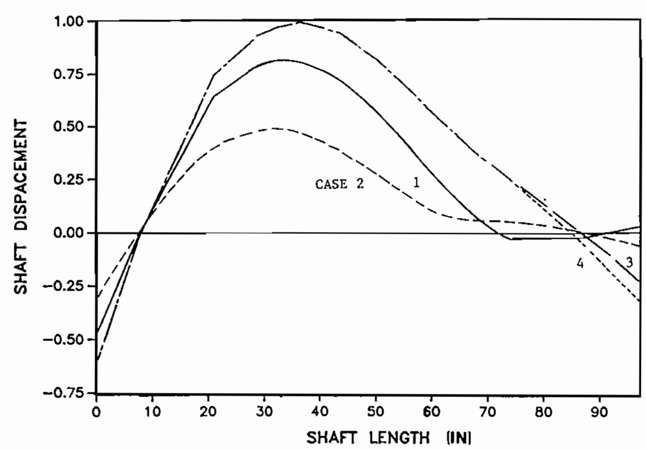

Figure 9. Mode Shapes—5000 RPM.

An unbalance response analysis was carried out for Cases 1, 2, 3, and 4 to clear up the question of the existence and location of critical speeds. Two equal unbalances of 1.0 oz in were located 90 degrees apart, one at midspan and one at a quarter span (Figure 3). The shaft displacement response near midspan is shown in Figure 10. The Case 1 critical speed shows up at about 5,000 rpm, as expected. Case 2 has the highest unbalance sensitivity with the critical speed at about 4,600 rpm, while Cases 3 and 4 have heavily damped critical speeds at about 4,500 rpm. The large difference in response between Cases 1, 2, and 3 shows that both moment coefficients and inertia coefficients have significant effects on rotordynamics. Also, the similarity between Case 3 and Case 4 responses indicates that the bearings have little effect on the rotor displacement in the current long, flexible rotor.

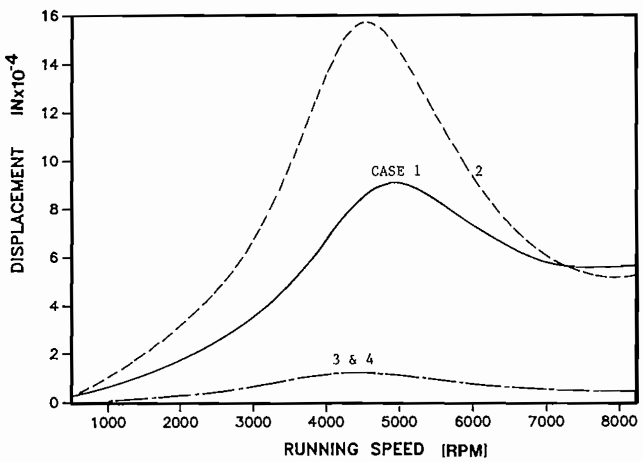

Figure 10. Shaft Displacement Response—Midspan.

Finally, the moment reaction at the balance piston is shown in Figure 11. The balance piston is the only seal in this pump whose moment contribution is significant. Case 1 obviously has no moment reaction because moment coefficients are neglected. Case 2 has the largest moment reaction and Case 3 and Case 4 have almost identical moment reactions, again demonstrating the significance of both the moment coefficients and the inertia coefficient and the insignificance of the bearings. It is expected that the bearing effect will be much more significant for a short, stiff pump rotor.

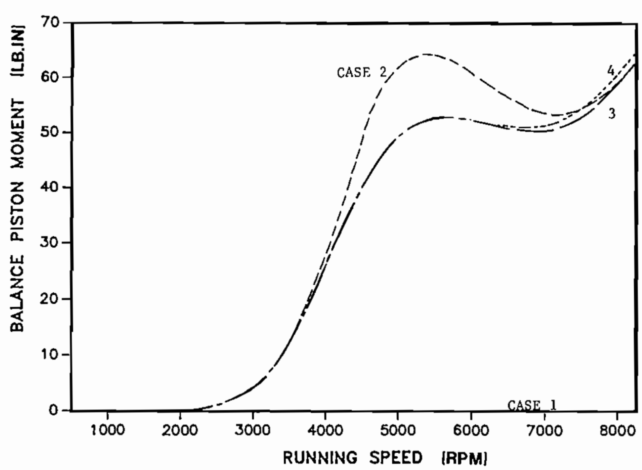

Figure 11. Balance Piston Moment Transmission.

One pump of the type analyzed had earlier been observed to have a subsynchronous whirl frequency at 4620 rpm when running at 6600 rpm [12]. This pump was analyzed as in Cases 1 and 4 to see what effect the new analysis capabilities have on the accuracy of the subsynchronous whirl prediction. With Case 1 options, the whirl frequency was predicted at 3970 rpm with a log dec of −1.8. With Case 4 options, the whirl frequency was 4290 rpm with a log dec of 1.2. Thus, the whirl frequency prediction was significantly improved using the Case 4 options, whereas the accuracy of the log dec prediction changes substantially.

SUMMARY

A new computer program for pump rotordynamic analysis has been developed with capabilities that surpass the prior state-of-the-art. The capabilities include the option for 4 × 4 seal stiffness, damping and inertia coefficient matrices, accurate calculation of bearing stiffness and damping coefficients for statically indeterminate rotors plus seal misalignments and bearing pedestal flexibilities.

The program has been used to analyze the dynamic characteristics of a commercial eleven stage centrifugal pump. The results showed that use of moment coefficients and inertia coefficients for the seals has a significant effect on the predicted dynamic characteristics of the pump rotor. Also, the prediction of a measured whirl frequency showed a significant improvement when these coefficients were incorporated.

The next phase of this work is to include an option for pump housing flexibility to permit accurate modelling of vertical pumps which often have very flexible housings.

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ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support provided for this work by the Turbomachinery Research Consortium at Texas A&M University.