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Impact of metal prices on plant optimization

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ABSTRACT

The optimum economic operating conditions of a processing plant are highly dependent on the market price of the metal produced. Since the price of a metal product is determined by supply and demand relationships and is greatly affected by local and world political events, future metal prices are not explicitly defined. Apart from historical trends, such factors create a degree of speculation. The study provides a method to account for the volatility of future metal prices when optimizing fully integrated mineral processing plants. A model was developed for estimating the "certainty-equivalent price" throughout the project lifetime while taking into account the volatility of future metal prices. The certainty-equivalent price should be used as a substitute for the volatile spot price in any optimization process that depends on the realized metal price. Non-ferrous metals were considered in the investigation rather than iron-bearing and/or precious metals. A hypothetical copper project was provided as an example to illustrate the applicability of the model and investigate the advantages of the suggested procedures in improving the financial performance of the plant.

Keywords: Metal; Mineral processing; Mineral economics; Process optimization

1. INTRODUCTION

Metals are important resources for many industries on a global basis. The enormous growth of industrialization in the past century has led to a significant increase in the demand for metals amongst established nations and developing countries. To meet the increasing demand, the annual output of many metals, particularly base metals, has grown dramatically (Wills, 1992). The excessive consumption of metals has resulted in the exhaustion of most rich ores, and consequently the potential profits from mining and processing operations have decreased due to the low grade of the available ores and the worldwide decline of most metal prices. In order for the mineral commodities industry to face the challenges, a sound management policy should be adapted that takes into account the unstable economic environment. In a world of continuously changing economic conditions, no plant operates under static conditions. At any point in time at least one of the interlocking components, which control the actual operation, is subject to change. For instance, ore grades fluctuate, market conditions vary, and new technology emerges (Evans, 1980).

The goal set for any processing plant is the production of a concentrate of the valuable mineral with a grade as high as possible and at a cost as low as possible. This goal is to be achieved simultaneously with maximizing the recovery (Currie, 1973). In order for a plant to realize the maximum economic performance, the concentrate grade and the recovery should be optimized in order to maximize the profit per unit of ore processed. This profit depends mainly on the price of the metal produced. As explained by Wills (1992), the price of most metals is governed by the supply and demand relationships and the prices of many metals, particularly copper, have not kept pace with inflation. Since the market prices of metals are determined by unforeseen economic and technological conditions, these prices are highly volatile both in short and long runs (McMillan and Speight, 2001). Under these market conditions, managers of processing plants have to set the optimum operating conditions of plants on the basis of the information available at the project start-up. These plants will continue in production, under the operating conditions that have been decided at the project start-up, for long periods in the future, governed by the estimated tonnage of ore reserves (measured and indicated) and the scheduled production rates. During this long period, the economic conditions, based on which the operating conditions are optimized, change continuously. The continuous

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adjustment of the operating plants according to the continuously changing economic conditions is neither feasible nor practicable, and in some cases uneconomical. On the other hand, the changing economic conditions may result in inefficient plant operations and in the worst case may result in uneconomical operations. To avoid these problems, the stochastic behavior of future metal prices should be accounted for when optimizing the plant at the project start-up.

This study provides a method to account for the volatility of metal prices when optimizing processing plants. An example for the potential applications of the proposed method is provided. This example illustrates how the suggested procedures can be applied to optimize the concentrate grade and the recovery of a hypothetical copper plant. Also, the paper illustrates the contribution of the suggested procedures to the improvement of the financial performance of the project.

2. OPTIMIZING CONCENTRATE GRADE AND RECOVERY UNDER UNCERTAINTY

2.1. Basic model

A processing plant is operating at the optimum economic conditions when it realizes the maximum revenue per ton of ore treated. The revenue per ton of ore processed is the product of the net smelter return per short ton of concentrate "NSR_c" and the weight of concentrate produced from one ton of feed "C". The "NSR_c" depends, among other factors, on the concentrate grade and the unit price of the metal produced. The weight of concentrate produced from one ton of ore depends on the concentrate grade and the recovery. Since there is an inverse relationship between the concentrate grade and the recovery, managers should select the point, at the grade-recovery curve, at which the revenue per ton of ore is maximized. The "NSR_c" can be expressed by the following equation (Farrish, 1989; Segovia and Schena, 1993):

$$NSR_c = 2000\frac{c_d}{100}\left(\frac{G-u}{100}\right)(P-r_c)-T_c-X+Y-C_r \tag{1}$$

where:

c_d percentage deduction,
G concentrate grade,
u unit deduction,
P unit price of metal,
r_c refining charge,
T_c treatment charge, per short ton of concentrate,
X penalties, per short ton of concentrate,
Y by-product credit, per short ton of concentrate, and
C_r realization cost, per short ton of concentrate.

The revenue per short ton of feed, F_r, is:

$$F_r = NSR_c C \tag{2}$$

where "C" is the weight of concentrate produced from one ton of ore, which can be expressed as:

$$C = R\frac{f}{G} \tag{3}$$

where "R" is the percentage recovery and "f" is the feed grade.

Substituting from Equation (3) into Equation (2), then:

$$F_r = NSR_c\frac{Rf}{G} \tag{4}$$

The grade-recovery relationship can be expressed such as (Hall, 1971):

$$G = M-\left(\frac{M-f}{100}\right)\left(\frac{AR}{100+A-R}\right) \tag{5}$$

where "M" is the pure mineral value, %, and "A" is the liberation coefficient. Substituting from Equation (5) into Equation (4), then:

$$F_r = \frac{NSR_c Rf}{M-\left(\frac{M-f}{100}\right)\left(\frac{AR}{100+A-R}\right)} \tag{6}$$

where the "NSR_c" is obtained from Equation (1) after substituting for the value "G" from Equation (5). At definite values of the variables "M", "f" and "A", Equation (6) can be easily solved to find the recovery "R" at which "F_r" is maximized. Substituting for this optimum recovery in Equation (5), provides the optimum concentrate grade.

2.2. Introducing uncertainty

As indicated from Equations (1) to (6) inclusive, both the optimum recovery and the optimum concentrate grade depend on the market price of metal "P", and some definitely known variables. The current metal price (the spot price at the decision time) is known, but future metal prices are unpredictable. They fluctuate over time, increasing and decreasing, depending on the behavior of future metal market. Using the current metal price in the optimization process is not appropriate. This implicitly assumes that the metal price is constant throughout the project life and definitely equals the current spot price. Such assumption ignores the uncertainty and risk associated with the future metal prices, which may make the underlying plant not operating at its maximum profitable potential. To overcome this problem, the risky future metal prices should be substituted by a constant certainty-equivalent price. This price represents the certainty-equivalent average price throughout the project life. Since this price is constant and risk-free, it could be inputted into Equations (1) to (6) to determine the optimum recovery and the optimum concentrate grade.

The first step in determining the certainty-equivalent price is to find an appropriate model describing the stochastic process of the metal price under study using the historical price data. The second step is to determine the certainty-equivalent price throughout the project life based on the price model.

2.2.1. Metal price modeling

The historical behavior of the majority of metal prices indicates that a stable linear trend does not exist to illustrate the major amount of metal prices. As explained by Krautkraemer (1998), metal price decreases by virtue of exploration and discovery, or technological change that lowers extraction cost. Eventually, the effect of increasing user cost outweighs the decrease in extraction cost, or exploration opportunities are exhausted, so that price begins to increase. Therefore, the metal prices are strongly related to long-run production costs. Pindyck (1991) argued that over the long-run, the price of a commodity like copper will follow a mean-reverting process, for which the mean reflects long-run marginal cost. Since the trend for metal prices is not stable, then metal prices can be modeled as mean-reverting process in which the real price of metal tends to revert back to a normal level. Dixit and Pindyck (1994) examined the historical price data of crude oil and copper using the unit root test and concluded that the prices are mean-reverting. It should be noted that the historical pricing employed in preparation of the calculations and resultant graphs is based upon American dollars. It is recognized that it was not practical to express currency in euros (€) as customary for the journal since this monetary system did not come into effect until the past decade.

In the mean-reverting process, the change in price "dP" over a small time interval "dt" can be represented as follows (Pindyck, 1991; Dixit and Pindyck, 1994; Dixit et al., 1999):

$$dP = \eta\left(\overline{P} - P\right)dt + \sigma\, dz \tag{7}$$

where η is the speed of reversion, $\overline{P}$ is the normal level of the price P, σ is the standard deviation of price changes and dz is the increment of a Wiener process ($dz = \varepsilon_t dt^{0.5}$ where ε_t is normally distributed with zero mean and unit standard deviation).

If the value of metal price is currently "P_0", then its expected value at any future time "t" is:

$$E[P_t] = \overline{P} + \left(P_0 - \overline{P}\right)e^{-\eta t} \tag{8}$$

The expected price change from year "t-1" to year "t" can be expressed such as:

$$E[dP] = E[P_t] - P_{t-1} = \overline{P} + \left(P_{t-1} - \overline{P}\right)e^{-\eta} - P_{t-1} \tag{9}$$

Rearranging Equation (9);

$$P_t - P_{t-1} = \overline{P}\left(1 - e^{-\eta}\right) + P_{t-1}\left(e^{-\eta} - 1\right) \tag{10}$$

Since both "$\overline{P}$"and "η" are constants for the metal price under study, then Equation (10) can be rewritten as:

$$P_t - P_{t-1} = a + bP_{t-1} \tag{11}$$

Where

$$a = \overline{P}\left(1 - e^{-\eta}\right) \tag{12}$$

and

$$b = \left(e^{-\eta} - 1\right) \tag{13}$$

The parameters "a" and "b" can be determined by regression analysis using the historical data of metal prices. After determining "a" and "b", the normal level of metal price "$\overline{P}$" and the speed of reversion "η" can be determined using Equations (12) and (13) as follows:

$$\overline{P} = -\frac{a}{b} \tag{14}$$

and

$$\eta = -\ln(1+b) \tag{15}$$

After determining "$\overline{P}$" and "η" for the price of metal under study, then Equation (8) can be used to forecast the future values of metal price throughout the project life.

2.2.2. Certainty-equivalent price

Assume that the current spot price is "P_o", then, the expected future price at any future time "t" is obtained from Equation (8). Since these expected future prices are risky, the present value "PV_r" of the stream of the risky prices over the project lifetime "T" is calculated using the risk-adjusted discount rate "r" as follows:

$$PV_r = \int_o^T P_t e^{-rt} dt \tag{16}$$

Substituting for "P_t" from Equation (8), then:

$$PV_r = \int_0^T \left(\overline{P} + \left(P_0 - \overline{P}\right)e^{-\eta t}\right)e^{-rt} dt \tag{17}$$

then

$$PV_r = \frac{\overline{P}}{r}\left(1 - e^{-rT}\right) + \frac{\left(P_0 - \overline{P}\right)}{\eta + r}\left(1 - e^{-(\eta+r)T}\right) \tag{18}$$

Assume that the certainty-equivalent constant price throughout the project life is "P^*". Since this price is risk-free, the present value "PV_c" of the stream of the risk-free prices over the project life is calculated using the safe, risk-free, discount rate "r_f", as:

$$PV_c = \int_0^T P^* e^{-r_f t} dt \tag{19}$$

then

$$PV_c = \frac{P^*}{r_f}\left(1 - e^{-r_f T}\right) \tag{20}$$

PV_c represents the risk-free amount of money that the investor is willing to accept as a substitute for the risky amount "PV_r". Since the price "P^*" represents the certainty-equivalent of risky future prices, the present value of the stream of the risk-free prices calculated at the risk-free discount rate must equal to the present value of the stream of the risky prices calculated at the risk-adjusted discount rate. Equating "PV_c" to "PV_r" produces:

$$\frac{P^*}{r_f}\left(1 - e^{-r_f T}\right) = \frac{\overline{P}}{r}\left(1 - e^{-rT}\right) + \frac{\left(P_0 - \overline{P}\right)}{\eta + r}\left(1 - e^{-(\eta+r)T}\right) \tag{21}$$

Rearranging:

$$P^* = \frac{\overline{P}\, r_f}{r}\left(\frac{1 - e^{-rT}}{1 - e^{-r_f T}}\right) + \frac{\left(P_0 - \overline{P}\right) r_f}{\eta + r}\left(\frac{1 - e^{-(\eta+r)T}}{1 - e^{-r_f T}}\right) \tag{22}$$

where

P^* certainty-equivalent constant price throughout the project life,
P_o current spot metal price at the decision time, year 0,
$\overline{P}$ normal level of metal price,
η speed of reversion,
T project lifetime, years,
r_f risk-free real discount rate, usually based on government bond rates (Smith, 1995), and
r risk-adjusted real discount rate.

The risk-adjusted real discount rate is determined using the capital asset pricing model as follows (Brealey and Mayers, 1996; Lumby and Jones, 1999):

$$r = r_f + \beta\left(r_m - r_f\right) \tag{23}$$

where

β project beta, which reflects the degree of responsiveness of the expected return on the project relative to movements in the expected return on the market. For example β= 1.13 for base metals mining and 0.27 for gold mining (Smith, 1995). Since most of project risk comes from the volatility of metal prices, the beta factor is assumed to be reflecting the variability of the metal prices with respect to the market,

r_m expected return on the market, and

r_m-r_f market risk premium.

The price "P^*" obtained from Equation (22) is the constant average price throughout the project life. Hence, it should be inputted into any optimization process that depends on the realized metal price.

3. DISCUSSIONS

Consider a copper project in which the copper ore is mined out and processed, and the copper is refined and sold by a single company. For simplicity, assume that the project produces only copper cathodes and does not produce any other by-products. The economic performance of the processing plant is measured by the net smelter return per ton of feed "F_r". The plant is operating at the optimum economic conditions when "F_r" is maximized. As indicated from Equations (1) to (6), "F_r" depends mainly on the concentrate grade, the recovery and the copper price. Since the size of the project is small with respect to the whole market, the project is a price-taker rather than a price-maker. Therefore, the price of copper is tied to the project and the managers of the project can do nothing with the copper price. They can only select the operating conditions of the plant (e.g. the concentrate grade and the recovery) so as to maximize "F_r" at the given copper price. Table 1 lists the prices of copper cathodes in 1998 constant American dollars for the period from 1921 to 2000*. (*The prices are obtained from U.S. Geological Survey Open-File Report 01-006 in $/metric tonne of copper cathodes and converted to USD/lb.)

Table 1
Real copper prices from 1921 to 2000 in 1998 constant US dollars

Year	Price, USD/lb	Year	Price, USD/lb	Year	Price, USD/lb	Year	Price, USD/lb
1921	1.153	1941	1.329	1961	1.647	1981	1.510
1922	1.316	1942	1.202	1962	1.665	1982	1.230
1923	1.407	1943	1.134	1963	1.652	1983	1.253
1924	1.266	1944	1.112	1964	1.702	1984	1.050
1925	1.325	1945	1.094	1965	1.824	1985	1.015
1926	1.289	1946	1.171	1966	1.810	1986	0.982
1927	1.221	1947	1.552	1967	1.860	1987	1.184
1928	1.411	1948	1.506	1968	1.933	1988	1.662
1929	1.747	1949	1.334	1969	2.109	1989	1.723
1930	1.298	1950	1.456	1970	2.439	1990	1.535
1931	0.898	1951	1.543	1971	2.098	1991	1.309
1932	0.690	1952	1.502	1972	2.005	1992	1.248
1933	0.917	1953	1.774	1973	2.183	1993	1.034
1934	1.057	1954	1.815	1974	2.555	1994	1.221
1935	1.057	1955	2.287	1975	1.944	1995	1.480
1936	1.143	1956	2.514	1976	1.993	1996	1.133
1937	1.520	1957	1.742	1977	1.797	1997	1.087
1938	1.184	1958	1.488	1978	1.645	1998	0.787
1939	1.316	1959	1.733	1979	2.071	1999	0.743
1940	1.343	1960	1.779	1980	2.005	2000	0.835

It is well known that there is an inverse relationship between the concentrate grade and the recovery, which is called the grade-recovery curve. Figure 1 shows the grade-recovery curve for a copper ore with a liberation coefficient of 23.0 and a pure mineral value of 47.37% [data obtained from Hall (1971)]. In order to maximize "F_r", the managers can move through the curve to the point that gives the maximum "F_r". Assume that the project is optimized in 1980 when the copper price was USD2.0/lb and the production commenced in 1981 at a capacity of 20,000 ton per day throughout the project life which is assumed to be 20 years. Also, assume that the refining cost was USD0.1/lb and the smelting cost was USD100/ton (Farrish, 1989). Based on these assumptions, the optimum recovery and the optimum concentrate grade that maximize "F_r" have been found to be 90.1% and 18.36%, respectively, as indicated in Figure 2. These optimum operating conditions depend on the copper price used in the analysis. If the copper price is changed, the optimum conditions will change. As illustrated in Figure 3, both the optimum recovery and the optimum concentrate grade are sensitive to the copper price used in the optimization process. As the copper price increases, the optimum recovery increases and the optimum concentrate grade decreases.

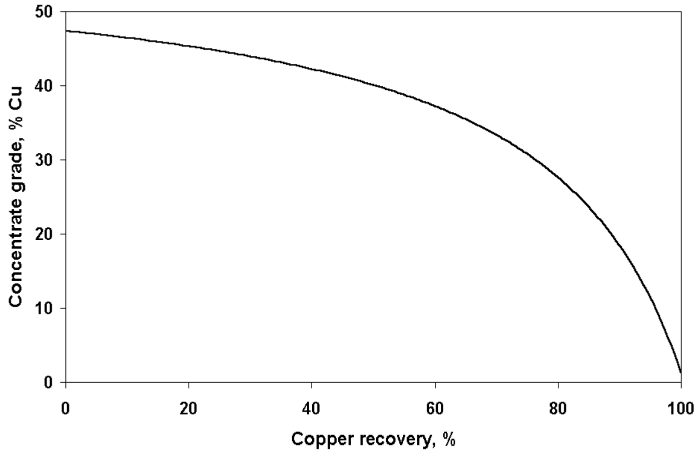

Figure 1. Grade-recovery relationship

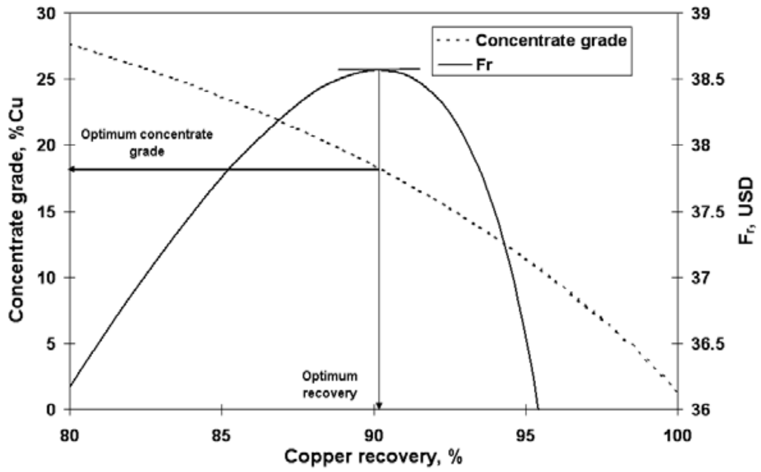

Figure 2. The optimum recovery and the optimum concentrate grade

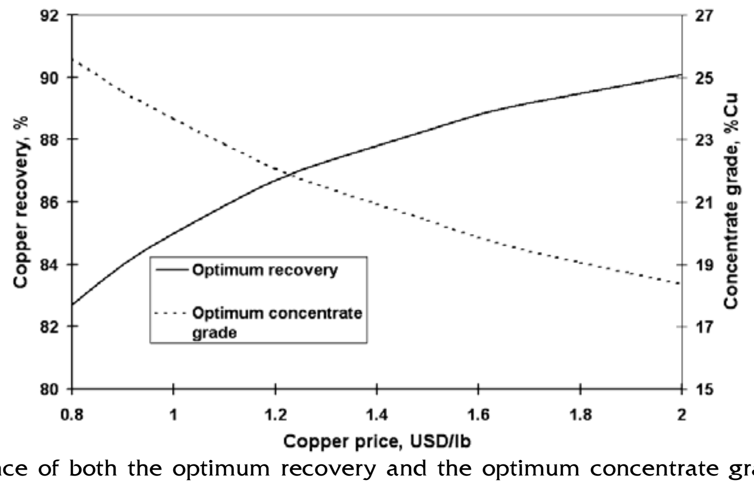

Figure 3. Dependence of both the optimum recovery and the optimum concentrate grade on the copper price

As shown in Figure 4, the copper price is highly volatile. It fluctuates from year to another due to unpredictable economic conditions. Therefore, managers of the plant should take into account such behavior of future copper prices when designing the plant. In this respect, the certainty-equivalent price throughout the project life should be estimated. This price represents the constant copper price upon which the metallurgical performance of the plant should be optimized.

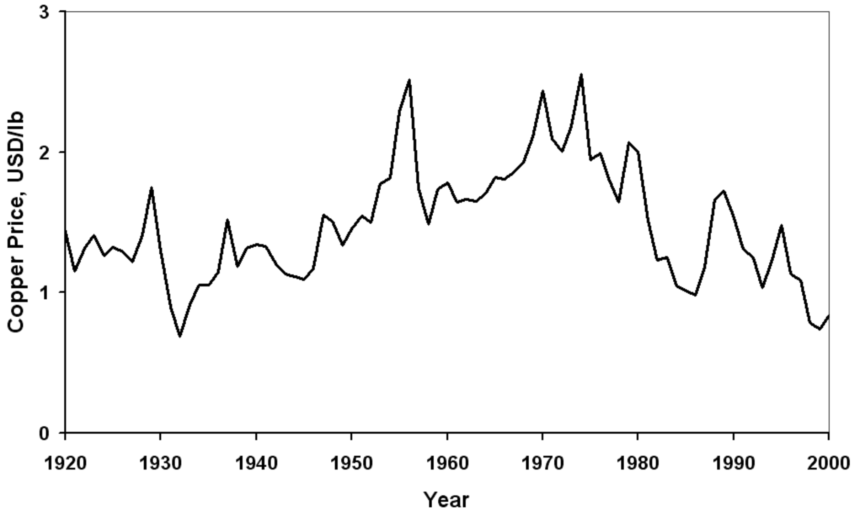

Figure 4. Copper price from 1921 to 2000 in 1998 constant US dollars

Based on the copper prices listed in Table 1, the normal level to which the copper price tends to revert and the speed of reversion have been found to be USD1.44/lb and 0.17 respectively. The certainty-equivalent price for our example project, using Equation (22) with a real-risk-free interest rate of 2.5%, a "β" factor of 1.13 and a market risk premium of 5% (Smith, 1995), has been found to be USD1.046/lb. Feeding it into the optimization process, the new optimum recovery and the optimum concentrate grade have been found to be 85.4% and 23.31% respectively. Figure 5 illustrates how the optimum recovery is changed when accounting for the fluctuation of future copper prices.

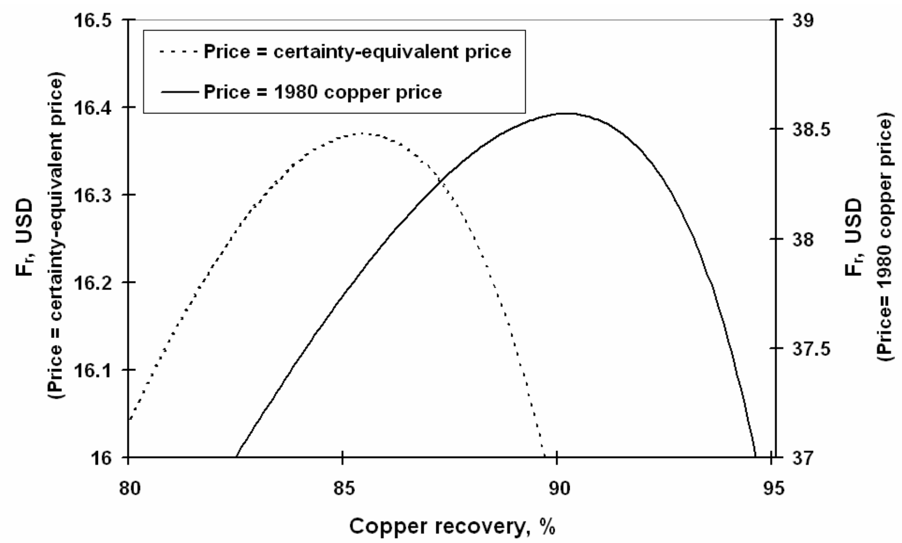

Figure 5. The change in the optimum recovery when accounting for the price volatility

It is important now to investigate the contribution of the procedures developed in this study in improving the financial performance of the project. Table 2 lists the actual copper prices throughout the life of the hypothetical project that is assumed to be optimized in the year 1980 and commenced production in 1981. The project is assumed to process 20,000 tons of copper ore per day throughout its 20 year life.

The annual revenues of the project are estimated based on the actual copper prices in the two cases, that is when optimizing the plant using the certainty-equivalent price and when optimizing the plant using the copper price prevailing at the project start-up (year 1980). Comparing the last two columns of Table 2, it is clear that the annual revenues generated when optimizing the plant using the certainty-equivalent price are mostly greater than the annual revenues generated when optimizing the plant using the price prevailing at the optimization time (year 1980). More definitely, optimizing the plant using the certainty-equivalent price has generated a USD38.44 million more revenue throughout the project life. Accordingly, it could be concluded that, optimizing the recovery and the concentrate grade using the certainty-equivalent price can improve the economic performance of the heavy industrial project.

Table 2
Comparison between annual revenues

Year	Actual copper price, USD/lb (1998 constant dollars)	Annual revenue when optimizing the plant using the certainty-equivalent price USDmillion	Annual revenue upon optimizing the plant using the 1980 price USDmillion
1981	1.510	187.958	188.710
1982	1.230	143.936	142.265
1983	1.253	147.505	146.030
1984	1.050	115.558	112.325
1985	1.015	110.197	106.669
1986	0.982	104.992	101.178
1987	1.184	136.673	134.602
1988	1.662	211.777	213.840
1989	1.723	221.415	224.008
1990	1.535	191.873	192.840
1991	1.309	156.357	155.369
1992	1.248	146.798	145.284
1993	1.034	113.058	109.687
1994	1.221	142.584	140.839
1995	1.480	183.242	183.734
1996	1.133	128.749	126.242
1997	1.087	121.406	118.495
1998	0.787	74.256	68.750
1999	0.743	67.401	61.518
2000	0.835	81.881	76.795

4. CONCLUSIONS

The optimum economic operating conditions of a processing plant are highly dependent on the price of the metal produced. The metal price based on which the plant is optimized is more likely to be changed because future metal prices are uncertain and that a long span of time exists between project commissioning and closing. Optimizing the plant on the basis of the metal price prevailing at the project start-up may result in inefficient plant operations.

The results indicated that both the optimum recovery and the optimum concentrate grade have been changed significantly when considering the volatility of future copper prices. Also, it has been found that the annual revenues generated from the project increase when optimizing the plant based on the certainty-equivalent price (P*) rather than the spot metal price prevailing at the optimization time.

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NOMENCLATURE

A liberation coefficient
C weight of concentrate produced from one ton of feed
c_d percentage deduction
C_r realization cost, per ton of concentrate
f feed grade
F_r revenue per ton of feed
G concentrate grade
M pure mineral value
NSR_c net smelter return per ton of concentrate
P unit price of metal
P^* certainty-equivalent constant price throughout the project lifetime
$\bar{P}$ normal level to which the metal price tends to revert
P_o current spot price of metal, at the project start-up, year 0
PV_c present value of a stream of certain prices over the project lifetime
PV_r the present value of a stream of uncertain prices over the project lifetime
R percentage recovery
r risk-adjusted discount rate
r_c refining charge
r_f safe risk-free discount rate
r_m expected return on the market
r_m-r_f market risk premium.
T project lifetime
T_c treatment charge, per short ton of concentrate
u unit deduction
USD United States dollars
X penalties, per short ton of concentrate
Y by-product credit, per ton short of concentrate
Greek letters
β the project beta, which reflects the degree of responsiveness of the expected return on the project relative to movements in the expected return on the market
η the speed of reversion