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Impact crushing approach to the relationship of energy and particle size in comminution

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ABSTRACT

The theories of Rittinger, Kick and Bond, that relate the energy required for comminution to the particle size of the feed and product do not provide for a size distribution of the material. The present theory tries to incorporate the concept of particle size distribution into the existing ones. Charles, firstly presented such an approach, but he used a different model of size reduction. He borrogh the model of Walker and Shaw according to which, the size of a particle is continuously reduced to an infinetly smaller particle at a time. The present model accepts that breakage of a mother paricle produces more than one daughter particles, which have a certain size distribution.

The energy required to create each individual particle, by exposing all of its surface, is given by the formula: $Q_x=(C_S.S_x)^n$ or $Q_x=(C_S.f)^n.x^{2n}$ where: x the size of the particle, S_x the surface area, f the surface coefficient, while C_S and n are constants >0. The specific energy (energy per unit mass) for the same created particle is: $q_x=(C_S.f)^n.x^{2n-3}/(k.\rho)$ where: ρ the particle density and k the volume coefficient. Assuming a GGS size distribution of the daughter particles, the energy required to produce an assembly of particles having a total mass W_o, size modulus y and distribution modulus α is: $E_y = \frac{(C_S \cdot f)^n \cdot \alpha \cdot W_o}{k \cdot \rho \cdot (2n-3+\alpha)} \cdot y^{2n-3}$,for (2n-3+α)> 0. In this case the laws of Rittinger and Bond are derived as partial cases for values of n equal to 1 and 1.25 respectively. For (2n-3+α) = 0 the energy is given by: $E_y = \frac{(C_S \cdot f)^n \cdot W_o}{k \cdot \rho} \cdot \frac{\ln y^{\alpha}}{y^{a}}$.

Keywords: Crushing; Comminution; Particle size; Energy

1. INTRODUCTION

As already known the basic theories that describe the relationship of energy and particle size in comminution are those of Rittinger, Bond and Kick given by equations (1), (2) and (3) respectively (Rittinger, 1867; Bond, 1952; Kick, 1885):

$$e_{R12} = C_R \cdot \left(\frac{1}{x_2} - \frac{1}{x_1} \right) \quad (1)$$

$$e_{B12} = C_B \cdot \left(\frac{1}{\sqrt{x_2}} - \frac{1}{\sqrt{x_1}} \right) \quad (2)$$

$$e_{K12} = C_K \cdot \log \frac{x_1}{x_2} \quad (3)$$

where, e_{12} the specific energy (energy per unit weight) for the reduction from initial size x_1 to final size x_2 and C=constant, the indexes R, B, K refer to the name of the respective theories. In the case of Bond the size corresponds to 80% passing.

In an effort to describe the comminution process by a general theory that will include the ones above as partial cases equation (4) was proposed by Walker and Shaw (1954).

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$$de_x = -C\frac{dx}{x^m} \tag{4}$$

where, de_x is the incremental specific energy required to reduce by dx the size of a particle of size x, while C and m are constants > 0 (Walker and Shaw, 1954). The integration of (4) for m=1 gives (5), which corresponds to Kick's law

$$e_x = -C \cdot \ln x \tag{5}$$

and the same integration for $m \neq 1$ gives (6), which corresponds to Rittinger and Bond laws, for m equal to 2 and 1.5 respectively.

$$e_x = \frac{C}{m-1} \cdot x^{1-m} \tag{6}$$

All the above approaches do not take into consideration the size distribution of the material. This was done by Charles, who uses equation (4) and assuming a GGS size distribution proposes that the energy required to break an initial particle of size x_m and produce an assembly of particles with a distribution modulus α and maximum size K is:

$$e_k = \int_0^K \int_{x_m}^x (-C \cdot dx_1 / x_1^m) \cdot (dW) \tag{7}$$

dW is defined bellow in (19) and the integration gives (Charles, 1957):

$$e_k = \frac{C \cdot \alpha}{(m-1) \cdot (1-m+\alpha)} \cdot K^{1-m} \tag{8}$$

Equation (8) gives the laws of Rittinger, and Bond as partial cases as (6) does. The difference is that (8) refers to a particulate material with sizes 0 to K while (4) refers to one size material.

Although equation (8) is an improvement compared to (6) it still depends on equation (4).The physical equivalent of (4), which is the basic equation used, is a proccess in which a particle is continuously reduced in size and no reference is made to the chips produced. For crushing and grinding the process is better represented by breakage events in which particles break into more than one smaller pieces (Lynch, 1977).

2. ANALYSIS OF THE MODEL

A new mathematical model describes the mechanism of impact crushing. The model is based on the following assumptions and definitions:

1. When a sufficient force is applied upon a particle, it breaks into smaller ones that all together keep the initial mass. Define x_c the smallest particle size that can exist, which is supposed to be a multiple of the molecule that exhibits the crystal lattice symmetry of the mineral, x a particle size in the assembly and y the largest particle size in this assembly. Compared to y the size x_c can be taken as zero.
2. The type of size distribution of the particles produced is kept constant for all crushing events and it is assumed to be the Gates, Gaudin, Schuhmann (GGS) described by equation (9)

$$W_x = W_o \left(\frac{x}{y}\right)^{\alpha} \tag{9}$$

where W_x the cumulative mass of particles smaller than x, W_o the total mass of the material, y the size modulus and α the distribution modulus.

3. In a crushing event, when a large particle breaks into smaller ones, the energy Q_x necessary to create a single particle of size x, by exposing all of its surface, depends on the surface area S_x of the new particle to a power n.

$$Q_x = (C_S \cdot S_x)^n \tag{10}$$

where C_S= constant >0 and by definition the specific energy q_x is:

$$q_x = \frac{Q_x}{M_x} \tag{11}$$

where M_x is the mass of the single particle.

The energy for the creation of the particles in the range dx around size x is:

$$dE_x = q_x \cdot dW_x \tag{12}$$

4. The surface area of a particle of size x is

$$S_x = f \cdot x^2 \tag{13}$$

and its mass

$M_x = k \cdot \rho \cdot x^3$ (14)

where f and k the surface and volume coefficients respectively and ρ the particle density.For sphetres f=π and k=π/6.

5. Assume that a particle of size x_p and mass W_o breaks into an assembly of smaller particles of size x_c to y_1, in this case one can define the following:

E_{y_1} = Energy required to create all the particles of size x_c to y_1

$$E_{y_1} = \int_{x_c}^{y_1} dE_x \tag{15}$$

dE_x is the incremental energy required to create the particles of the assembly in the size range dx around x.

E_p = Energy required to create the initial particle of size x_p

E_{p,y_1} = Energy required to break the particle of size x_p and create the assembly of particles of size x_c to y_1. By definition one has:

$$E_{p,y_1} = E_{y_1} - E_p \tag{16}$$

Since E_p is very small compared to E_y one can accept that

$$E_{p,y_1} = E_{y_1} \tag{16a}$$

6. In accordance with the above the energy of breaking the same particle of size x_p to an other assembly of particles of size x_c to y_2 is:

$$E_{p,y_2} = E_{y_2} - E_p \tag{16b}$$

And the energy E_{y_1,y_2} to break the assembly x_c to y_1 and produce the new one x_c to y_2, where $y_2<y_1$ is calculated from the difference of (16b) and (16):

$$E_{y_1,y_2} = E_{y_2} - E_{y_1} \tag{17}$$

or

$$E_{y_1,y_2} = \int_{x_c}^{y_2} dE_x - \int_{x_c}^{y_1} dE_x = \int_{y_1}^{y_2} dE_x \tag{17a}$$

3. SYNTHESIS OF THE MODEL

Using (10) and (13) the energy to create or detach a single particle of size x becomes:

$$Q_x = (C_S \cdot f)^n \cdot x^{2n} \tag{18}$$

From (11), (14) and (18) the specific energy is:

$$q_x = \frac{(C_S \cdot f)^n \cdot x^{2n}}{k \cdot \rho \cdot x^3} = \frac{(C_S \cdot f)^n}{k \cdot \rho} \cdot x^{2n-3} \tag{19}$$

for $A=(C_S \cdot f)^n/k \cdot \rho$, then

$$q_x = A \cdot x^{2n-3} \tag{19a}$$

The derivative of (9) gives the mass of particles in a range dx around size x.

$$dW_x = \alpha \cdot W_o \cdot \frac{x^{\alpha-1}}{y^\alpha} \cdot dx \tag{20}$$

Substituting (19) and (20) into (12) one obtains:

$$dE_x = \frac{\alpha \cdot (C_S \cdot f)^n \cdot W_o}{k \cdot \rho \cdot y^\alpha} \cdot x^{2n-4+\alpha} \cdot dx \tag{21}$$

Substituting (21) into (17a) and integrating from y_1 to y_2 for $(2n-4+\alpha \neq -1)$ or $(2n-3+\alpha \neq 0)$ the energy is found to be:

$$E_{y_1,y_2} = \frac{\alpha \cdot (C_S \cdot f)^n \cdot W_o}{k \cdot \rho \cdot (2n-3+\alpha)} (y_2^{2n-3} - y_1^{2n-3}) \quad (22)$$

The specific energy e_{y_1,y_2} is obtained by dividing the energy by the mass W_o of the material, which remains constant during comminution:

$$e_{y_1,y_2} = \frac{\alpha \cdot (C_S \cdot f)^n}{k \cdot \rho \cdot (2n-3+\alpha)} (y_2^{2n-3} - y_1^{2n-3}) \quad (23)$$

or

$$e_{y_1,y_2} = A \cdot \frac{a}{2n-3+\alpha} \cdot (y_2^{2n-3} - y_1^{2n-3}) \quad (23a)$$

The laws of Rittinger and Bond can be derived for n=1 and n=1.25 respectively.

For $(2n-4+\alpha) = -1$ or $(2n-3+\alpha) = 0$ the integration of 21 gives:

$$E_{y_1,y_2} = \frac{(C_S \cdot f)^n \cdot W_o}{k \cdot \rho} \left(\frac{\ln y_2^a}{y_2^a} - \frac{\ln y_1^a}{y_1^a}\right) \quad (24)$$

Dividing by W_o the specific energy becomes:

$$e_{y_1,y_2} = \frac{(C_S \cdot f)^n}{k \cdot \rho} \left(\frac{\ln y_2^a}{y_2^a} - \frac{\ln y_1^a}{y_1^a}\right) \quad (25)$$

or

$$e_{y_1,y_2} = A \cdot \left(\frac{\ln y_2^a}{y_2^a} - \frac{\ln y_1^a}{y_1^a}\right) \quad (25a)$$

If y_2 is sufficiently smaller than y_1, which is the usual case in milling, then equations (23a) and (25a) are simplified to (26) and (27) respectively:

$$e_y = A \cdot \frac{\alpha}{2n-3+\alpha} \cdot y^{2n-3} \quad \text{for} \quad (2n-3+\alpha = 0) \quad (26)$$

$$e_y = A \cdot \frac{\ln y^\alpha}{y^\alpha} \quad \text{for} \quad (2n-3+\alpha \neq 0) \quad (27)$$

Where: y is the size modulus of the material produced when the energy input per unit weight is e_y, while n and α as defined above.

4. EXPERIMENTAL DATA

Experimental data from the literature, which relate the energy to the particle size and can be used to calculate the parameters n and α, are presented in Table 1.

Table 1
Experimental data from the literature

Mineral	2n-3	n	α	2n-3 +α	Source
Talc	-0.27	1.37	0.27	0.00	Hicom Pty, 2001
Cement	-0.32	1.34	0.35	0.03	Charles, 1957
Anhydrite	-0.50	1.25	0.59	0.09	Hicom Pty, 2001
Calcite	-0.51	1.25	0.46	-0.05	Hicom Pty, 2001
Magnesite	-.55	1.23	0.60	0.05	Tsiboukis, 1995
Flat glass	-0.62	1.19	0.64	0.02	Hicom Pty, 2001
Serpentine	-0.66	1.17	0.66	0.00	Hicom Pty, 2001
Sulfide ore	-0.73	1.14	0.69	-0.04	Kambossos, 1997
Pumice	-0.89	1.06	0.88	-0.01	Hicom Pty, 2001
Quartz	-0.90	1.05	0.91	0.01	Charles, 1957
Coal	-0.91	1.05	1.00	0.09	Charles, 1957
Fluorite	-0.99	1.01	1.01	0.02	Charles, 1957

continued

Mineral	2n-3	n	α	2n-3 +α	Source
Pyrex glass	-1.07	0.97	1.09	0.02	Charles, 1957
Salt, NaCl	-0.76	1.12	1,17	0.41	Charles, 1957
Galena	-0.82	1.09	1.00	0.18	Charles, 1957
Pyrex glass	-1.38	0.81*	1.40	0.02	Charles, 1957

* high impact

Values of α (distribution modulus) are calculated from size distribution measurements of comminution products by plotting the weight % passing as a function of size in log-log diagrams. The value of α is the slope of the straight line. From the same diagrams the value of y (size modulus) for the particulate material is also determined. The value of y corresponds to the size at which the straight line intersepts the 100% passing level. The exponent (2n-3) can be calculated from the log-log plot of energy consumed to produce a particulate material versus the size modulus and it is the slope of the corresponding line. There are different kinds of tests used that relate the energy to the particle size, such as:

- Frictionless crusher (Hukki, 1943), Drop weight or Projectile Impact tests (Charles, 1957), Compressive loading (Tavares-King ,1998) e.t.c.
- Rod-mill/ball mill tests assume the energy used for breaking the particles is a constant proportion of that consumed by the mill. In these tests one can either relate the size modulus to the energy used (Charles 1957), (Tsiboukis, 1996) or relate the production rate of each particle size to the energy since the specific energy is inversely proportional to the production rate (Bond, 1952; Patronis, 1992).

The quantity (2n-3+α) in equations (23) and (26) is positive, because comminution is an energy consuming reaction (endothermic), thus (2n-3+α) >0 or the same α > -(2n-3). The experimental value of (2n-3+α) is calculated indirectly as the difference of the two measured quantities, (2n-3) and α, consequently the experimental error is accumulated in the calculation of (2n-3+α). The calculated values of (2n-3+α) from Table 1 are very small and some appear negative, as a result of experimental error. For some minerals this quantity is well above zero indicating that equations (23) and (26) are valid. However for most minerals (2n-3+α) is very small and practically can be taken as zero indicating that equations (24) and (27) should be used.

5. CONCLUSIONS

Equations (19a) and (26), given above, are of the same form. The first one gives the specific energy (energy per unit mass) required to expose the surface of a particle of size x, while the second gives the specific energy of a particulate material of size modulus y. If for example x = y, it is obvious that the energy per unit mass required to produce equidimensional particles is much less than the corresponding energy for a material of the same size modulus which consists of grains distributed in size.

Equations (19a) and (26), can also be used to derive Rittinger, Bond and Kick laws for values of n equal to 1, 1.25 and 1.5 respectively. It is reminded here that according to Kick the energy required to break a particle is proportional to its mass and consequently the specific energy is constant and does not depend on the size.

Equation (26), derived from the present model, is the same as equation (8) given by Charles and can be used in the same way. As mentioned at the beginning, the difference is that in the present model equation (19) was used instead of (4). In this model it is assumed that when a particle diminishes in size it breaks to more than one piece. On the contrary in the model presented by Walker and Shaw and adopted by Charles, when a particle diminishes in size the size reduction is continuous without breaking into pieces.

The quantity (2n-3+α), of the present model, used in equations (23) and (26) is the same as the quantity (1-m+α), proposed by Charles, in equation (8). As indicated by the experimental data of Table 1, many of which are given by Charles, for most minerals (2n-3+α) = (1-m+α) ≈ 0 and only for a few ones (2n-3+α) = (1-m+α) > 0. As Charles has mentioned, the minerals for which (1-m+α) is well above zero are the ones that exhibit cleavage in their structure, such as galena and salt (Charles, 1957), while for the rest of the minerals tested, (1-m+α) is almost zero. Consequently, the integration of equation (7) is not valid and one should have used the logarithmic formula of integration. In the present work the appropriate integration is used and equations (24) and (27) are produced which apply for (2n-3+α) = (1-m+α) ≈ 0. As mentioned before, this is the case for most minerals with no cleavage.

Most of the experimental data fit a curve of the form (26), $e_y = \beta.y^{2n-3}$, which gives a straight line in a log-log plot, of e_y versus y, $\log(e_y) = \log(\beta) + (2n-3)\cdot\log(y)$, and is used to calculate (2n-3).

The paradox is that, although equation (26) is used to describe the data, the quantity (2n-3+α) measured indicates that (26) is not valid for most data because (2n-3+α)≈0 and equation (27) should have been used instead.

An analysis of the mathematical forms of (26) and (27) is helpful at this point. For $(2n-3+\alpha) = 0$, any equation of the form $\sigma(y) = y^{(2n-3)}$ can be written as $\sigma(y) = 1/y^{\alpha}$. Now assume an equation of the form $\varphi(y) = \ln(y^{\alpha})/y^{\alpha}$ and calculate the difference $\varphi(y) - \sigma(y)$ shown is Figure 1, plotted for $\alpha = 1$, 0.5 and 0. It is obvious from Figure 1 that practically, for values of y several times the unit of measurement, both $\varphi(y) = \ln(y^{\alpha})/y^{\alpha}$ and $\sigma(y) = 1/y^{\alpha}$ give the same value since their difference tends to zero. In a log-log diagram equation $\sigma(y) = 1/y^{\alpha}$ gives a straight line of slope $-\alpha$, as shown in Figure 2, plotted for $\alpha = 1$. Equation $\varphi(y) = \ln(y^{\alpha})/y^{\alpha}$ is also plotted in Figure 2, for $\alpha = 1$. It is obvious that $\sigma(y)$ is always a positive straight line, but $\varphi(y)$ tends to zero for values of y approaching unit, while for $y<1$, $\varphi(y)$ becomes negative. Nevertheless, for values of y several times larger than the unit of measurement, $\varphi(y)$ is practically a straight line with slope very close to $-\alpha$. In this area, both lines are practically parallel and experimental data can be expressed by both equations with the proper selection of β and the size unit. Practically equation (26) can be used to determine the slope $(2n-3)$, also for the cases where $(2n-3+\alpha)=0$. Similar conclusions are also drawn, by accepting the model of Walker and Shaw for size reduction (Stamboliadis, 2002).

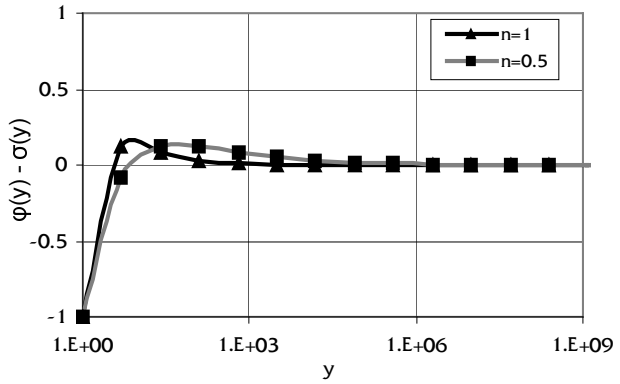

Figure 1. Plot of $\varphi(y) - \sigma(y)$

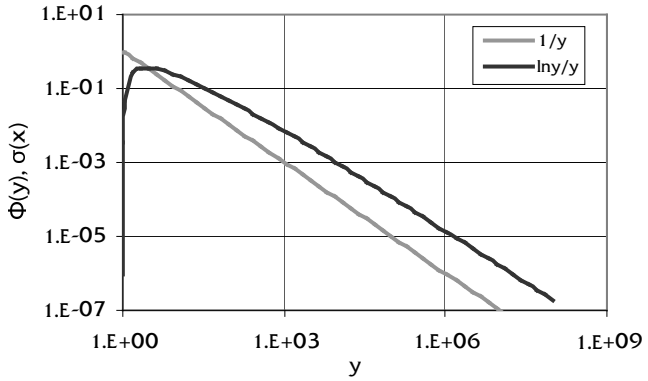

Figure 2. Plot of $\varphi(y)$ and $\sigma(y)$

An other important point is that for most minerals $\alpha = -(2n-3)$ since $(2n-3+\alpha) = 0$. For any particle range for which, n, is constant, the slope, α, is also constant and the size distribution of the material can be defined by any point in the $\log(W_x) - \log(x)$ diagram. To be more specific, for a material that obeys Bond's law $n=1.25$ and $\alpha = 0.5$; consequently the size distribution of this material can be defined by any point in the diagram $\log(W_{ax}) - \log(x)$ because the slope is defined. It is clear that Bond's proposal to define the size of a particulate material by the 80% passing size is correct, because there is only one distribution line that passes from the specific point, although it sounds arbitrary to some other people (Lynch, 1977). Actually any size can be selected to define the material in the particular size range since the slope is $\alpha = 0.5$. In the case of a material that obeys Rittinger's law the slope is $\alpha = 1$ and again the same situation holds.

As its has been explicitly explained by Hukki the exponent $(2n-3)$ can vary from one size range to the other depending on the mechanism of breakage, which determines whether the material breaks through crystal boundaries or the crystals themselves are broken (Hukki, 1961). It is expected that both, n, and α, depend on the size range of the material but are more or less constant in the same size range.

NOMENCLATURE

α, Distribution modulus of a particulate material
A, β Constants defined in the text
ρ, Particle density
C_{index}, Constants according to index
$e_y = E_y / Wo$, Specific energy of a particulate material of size modulus y
E_y, Energy of a particulate material of size modulus y
$E_{y1,y2}$, Energy to go from a particulate material of size y_1 to another y_2
f, Surface coefficient
k, Volume coefficient
m, Exponent
$M_x = k.\rho.x^3$, Mass of a particle of size x
n, Exponent
$q_x = Q_x / M_x$, Specific energy of a single particle
Q_x, Energy of a single particle
$S_x = f.x^2$, Surface of a particle
$V_x = k.x^3$, Volume of a particle
W_o, Total mass of a particulate material
W_x, Mass of a particulate material finer than size x,
x, particle size
y, Size modulus of a particulate material (max. size for GGS distribution)

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