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General gamma representation for product particle split in gravity concentrators

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ABSTRACT

This paper attempts the well-known regularized gamma distribution function to represent the separation efficiency of particle segregation in gravity concentrators. The model has four parameters that are evaluated by a least square fit between the model estimate and the measured data. The model takes into account the bivariate effects of particle size and particle density to define the size-density separation efficiency, typically recognized as the partition surface. The model derivation is rooted in the observed pivot phenomenon associated with size-based partition curves of the separators. Although, the mathematical representation is empirical in nature, it is generic and is applicable to various gravity units notwithstanding differences in equipment design and particle flow profiles in the gravity units. The suitability of the representation is examined using several sets of measured data from literature. Convenient functional forms for computing the size dependent separation indices namely cut density and Ecart probable using the model parameters have been proposed.

Keywords: Particle size-density separation; Partition surface; Gravity concentrators; Regularized gamma function

1. INTRODUCTION

Gravity separators of various designs treat vast tonnage of coal and mineral ores, typically in the size range of 0.01mm to 100mm. In gravity concentrators, various mechanisms such as intermittent fluidization; cross flow and vertical stratification in thin flowing-films; momentary jerks on separating particles; shear, centrifugal, viscous and pneumatic forces or a combination of these forces along with the gravity force influence separation of particles. In addition, the particle-particle and the particle-fluid interactions originating from the particle flow in the separator randomize the movement of particles and disperse them thereby enforcing a probabilistic effect on particle segregation as opposed to the ideal perfect separation.

It is normal practice to assess the performance of particle segregation in separators in terms of separation efficiency defined as the probability of particle split of specified attribute(s) (say, size and/ or density) to one of the product streams. This paper considers separation efficiency as split of particles of specified attribute(s) to the sink stream. The separation efficiency of classifiers is generally measured in terms of particle size while those of gravity concentrators in terms of particle density. These separation efficiencies can be conveniently represented by a few generic parametric equations as discussed in literature. The separation efficiency of separating units depends on the operating and design variables of the unit and thus there exists the scope for the improvement of the unit's performance by tuning the operational or design variables. An advantage of the separation efficiency curves is that when coupled with feed distribution they yield product particle distributions (King, 2001).

Although, in usual practice the separation efficiency of gravity concentrators is expressed in terms of particle density, a closer look at the effect of particle size on separation reveals gradual fall in separation efficiency curves with decreasing particle size. A simultaneous consideration of the effects of both particle size and particle density on separation efficiencies necessitates a bivariate parametric representation, typically referred to as partition surface hereafter. The advantage of having such a representation helps to monitor bivariate product distributions by coupling partition surface with the bivariate feed washability data expressed in terms of particle size and particle density.

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Ferrara and Schena (1987) reported a 3-dimensional representation of the general partition surface, under ideal conditions of stable medium suspension for dense media concentrators. It has been observed that the same representation holds good for particle separations in other types of gravity concentrators also. Any point on the partition surface represents a partition number, which corresponds to the fraction of feed of given size and density reporting to the sink stream. Figure 1 shows an illustrative plot of partition surface generated by the gamma model, discussed later in the text. Figure 1 can be re-plotted into two-dimensional size-based or density-based families of curves by sectioning the partition surface by planes normal to the size axis and the density axis respectively. Figures 2 and 3 respectively illustrate density-based and size-based families of curves generated from Figure 1. Figure 2 features reverse classification of particles for those particles whose particle density is lesser than the pivot density, ρ_p, while Figure 3 features pivoting of size-based partition curves at the pivot point characterized by the pivot density ρ_p and the pivot partition number Y_p. In addition, Figure 2 shows flattening of density-based curves to a partition number Y_p as the separation particle size approaches zero. In other words, it reveals that very fine particles in the separator split with a constant probability of Y_p irrespective of particle density, which can be referred to as by-pass fraction similar to the classification terminology.

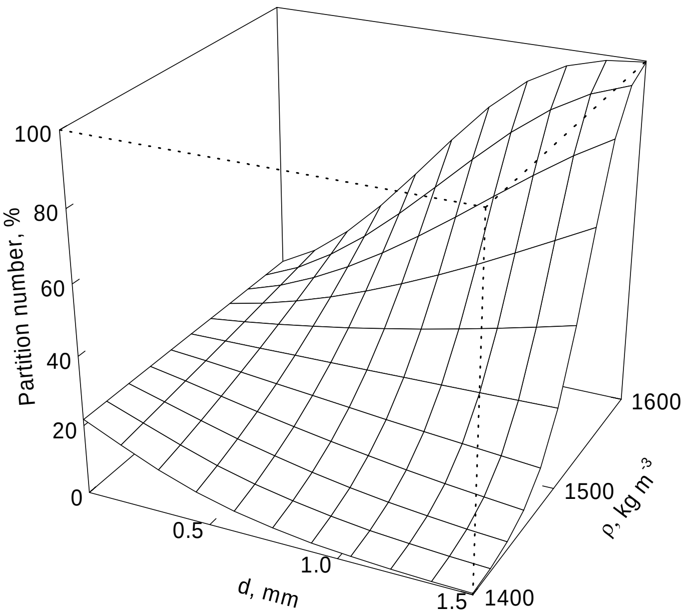

Figure 1. A typical representation of the partition surface generated from gamma model, Eq. (7) with parameters: ρ_p =1497, a =2.181, u = 20.099 and v =1.132.

2. PREVAILING MODELS

Single particle attribute mathematical models due to Lynch and Rao (1968) and Plitt (1971) represented by Eqs. (1) and (2) are employed to assess the performance of gravity concentrators.

$$Y = \frac{\exp(\alpha x) - 1}{\exp(\alpha x) + \exp(\alpha) - 2} \tag{1}$$

$$Y = 1 - \exp\left[-x^m \ln(2)\right] \tag{2}$$

where Y is fraction of feed reporting to sink, $x = \rho / \rho_{50}$ is ratio of particle density ρ to cut density ρ_{50}, and parameters α and m reveal the sharpness of separation.

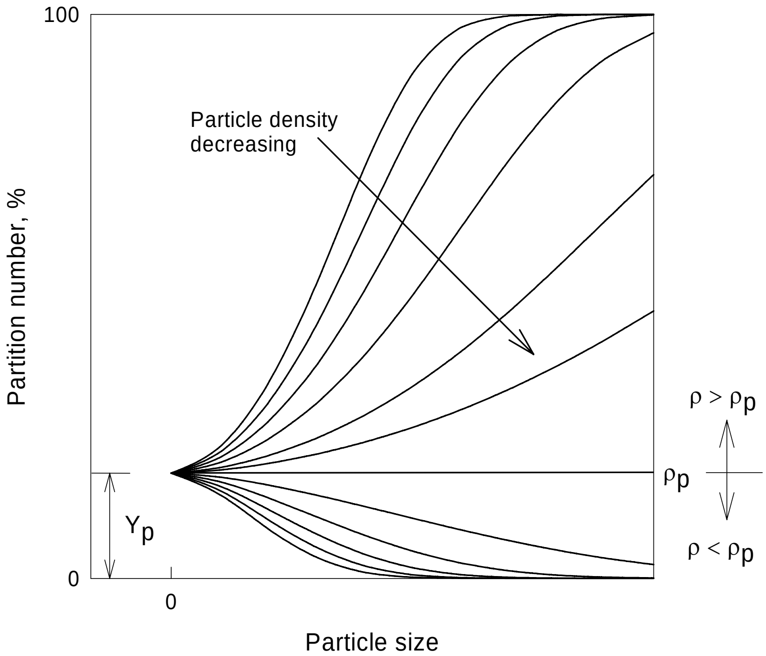

Figure 2. Density-based classification curves for different particle densities, generated from Figure 1

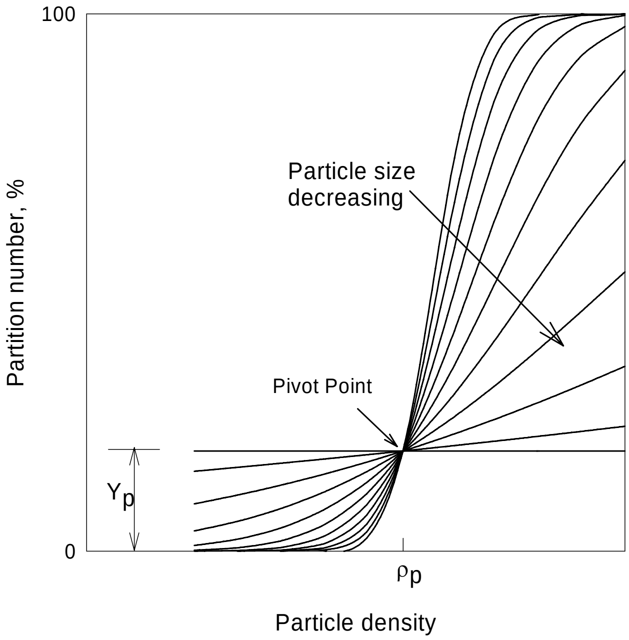

Figure 3. Size-based classification curves for different particle sizes, generated from Figure 1

A direct extension from single attribute (say, density) representation to bi-attribute (density and size) representation render Lynch and Rao and Plitt models inadequate to describe the size-density partition surface, as this requires knowledge of ρ_{50} value for each size or size-class. Lynch and Napier-Munn (1986) and Scott and Napier-Munn (1992) got around this difficulty by extending Lynch and Rao model under the constraints of observed pivot phenomenon. Their model is shown in Eq. (3). Klima and Luckie (1989) also have independently proposed Eq. (3).

$$Y = \frac{100}{1 + \exp[\ln(Y_p^{-1} - 1) + 1.099(\rho_p - \rho)/(k.d^n)]} \tag{3}$$

The model parameters Y_p, ρ_p, k and n are estimated using a least square fit between measured data and the model estimate. It has been argued that the parameter k incorporates the effect of viscosity while the parameter n incorporates the degree of turbulence within the separator.

Recently, Venkoba Rao *et al.* (2003a,b) proposed an elegant stochastic model for size-density partitioning of particles in gravity concentrators by considering a random walk on settling particles that are resisted by the drifting fluid within the separator. This derivation is in line with the suggestions of Kelly and Subasinghe (1991) for incorporating particle settling-velocity to describe separation efficiency. As per stochastic model, steady state split of particles to sink stream is given by

$$Y = 50\left[1 - erf\left(Ad^c(\rho - \rho_p) - B\right)\right] \tag{4}$$

where A, B, c and ρ_p are model parameters. The parameter A incorporates strength of centrifugal and viscous forces while the parameter B reveals strength of drifting fluid. The value of parameter c indicates the degree of turbulence in the separator. Pivot phenomenon that develops out of the model distinguishes stochastic model from the other proposed models. Analytical expressions for cut density and Ecart probable of stochastic model are in agreement with the empirical relations found in literature. Moreover, it is possible to derive Scott and Napier-Munn model from the stochastic model.

3. CURRENT WORK

In the present work, a regularized gamma function is proposed to represent partition surface of gravity concentrators. The derivation depends on the pivot phenomenon of size-based partition curves. A regularized incomplete gamma function given in Eq. (5), represents partition curve similar to a Weibull partition curve (refer Eq. (2)) where the particle attribute z varies between 0 and ∞.

$$Y = \frac{\int_0^z t^{a-1} \exp[-t]dt}{\int_0^\infty t^{a-1} \exp[-t]dt} = \frac{\gamma(a,z)}{\Gamma(a)} \quad and \quad a > 0 \tag{5}$$

Equation (5) can be modified to obtain the partition surface by considering z as a function of particle size and particle density. In addition, it is necessary that the functional form of z satisfies the pivot phenomenon of size-based partition curves at $\rho = \rho_p$. Analyses of measured data suggests that the power law in particle size satisfies the nessary constraint of pivot phenomenon. The functional form of z thus conforms to

$$z = \left[\frac{\rho}{\rho_p}\right]^{\left(ud^v\right)} \tag{6}$$

Combining Eqs. (5) and (6), and expressing the partition numbers in percent, the gamma model in standard notation is written as

$$Y = 100 \frac{\gamma\left(a, \left[\frac{\rho}{\rho_p}\right]^{\left(ud^v\right)}\right)}{\Gamma(a)} \tag{7}$$

where the model parameters a, ρ_p, u and v are estimated from a least square fit of the model estimation with measured partition coefficients. The parameters u and v define the sharpness and flatness of the surface. The parameters u and v respectively account for the strength of viscous forces and turbulence in the separator. The pivot partition number that represents the by-pass fraction in gravity concentrators (with regard to size attribute) can be computed from the gamma model by imposing $\rho = \rho_p$ in Eq. (7) which yields;

$$Y_p = 100\frac{\gamma(a,1)}{\Gamma(a)} \tag{8}$$

4. VALIDATION OF PROPOSED MODEL

The proposed gamma model is validated with 29 sets of data from literature for various gravity concentrators. The data are taken from the works of Miller (1969), Deurbrouck and Hudy (1972), Deurbrouck and Palowitch (1979), Llewellyn *et al.* (1979), Palowitch and Deurbrouck (1979), Collins *et al.* (1983), King and Juckes (1984), Lynch and Napier-Munn (1986), Ferrara and Schena (1987), Scott *et al.* (1987), Apodaca (1988), Kelly *et al.* (1988), Nicol and Bensley (1988), Restarick and Krnic (1991), Scott and Napier-Munn (1992), Atesok *et al.* (1993), Honaker *et al.* (2000) and Galvin *et al.* (2002). Levenberg-Marquardt least-square minimization technique is used to obtain best-fit model parameters, which are tabulated in Table 1 along with the sum of squared errors (SSE). Figure 4(a, b, c, d, e, f) illustrates a comparison of model generated partition curves with measured partition data for a few gravity concentrators. The deviation between measured data and the model fit is attributed to inherent experimental and analyses errors.

Table 1
Summary of gamma partition surface parameters along with sum of squared errors (SSE) for various gravity concentrators

Sl. No.	Separator type	Gamma Model					Reference
		a	ρ_p	u	v	SSE *	
1	Dense Medium Cyclone	2.181	1497	20.099	1.132	0.314	Survey 1.1 of Scott
2	Dense Medium Cyclone	1.593	1455	15.267	1.096	0.372	Survey 1.2 of Scott
3	Dense Medium Cyclone	1.069	1432	15.575	1.060	0.300	Survey 1.3 of Scott
4	Dense Medium Cyclone	2.149	1564	7.712	1.068	0.252	Survey 2.1 of Scott
5	Dense Medium Cyclone	1.778	1555	8.946	0.936	0.218	Survey 2.2 of Scott
6	Dense Medium Cyclone	1.203	1554	8.374	1.075	0.136	Survey 2.3 of Scott
7	Dense Medium Cyclone	1.811	1336	2.025	1.362	0.279	Survey 3.1 of Scott
8	Dense Medium Cyclone	1.886	1403	32.757	0.568	0.052	King & Juckes
9	VORSYL separator	2.295	2764	8.123	0.435	0.092	Fig 9 of Collins et al
10	Dense Medium Cyclone	1.400	1220	51.204	0.821	0.040	Restarick & Krnic
11	Dense Medium Cyclone	2.807	3001	8.303	0.959	0.172	Lynch & Napier-Munn
12	Chance Cone Separator	2.678	1331	2.558	0.769	0.039	Palowitch & Deurbrouck
13	Dense Medium Cyclone	2.284	1307	3.442	0.593	0.064	Palowitch & Deurbrouck
14	TRI FLO separator	1.154	2429	92.107	1.784	0.055	Fig 8 of Ferrara & Schena
15	TRI FLO separator	1.142	2558	19.803	1.126	0.068	Fig 9 of Ferrara & Schena
16	Media densifying Cyclone	2.261	3629	24.600	0.708	0.020	Fig 10 of Ferrara & Schena
17	Reflux Classifier	8.559	1154	6.159	0.445	0.084	Galvin et al
18	Air Table	6.568	737	1.182	0.382	0.063	Llewellyn et al
19	Wemco drum separator	1.430	2849	0.998	0.694	0.253	Scott et al
20	Feldspar Jig	9.501	964	3.767	0.181	0.139	Deurbrouck & Palowitch
21	Richert Spiral	2.199	1516	7.835	0.601	0.244	Fig 1 of Atesok et al
22	Richert Spiral	2.222	1461	10.842	0.570	0.117	Fig 2 of Atesok et al
23	Shaking Table	2.786	1398	8.095	0.310	0.420	Deurbrouck & Palowitch
24	Humphreys Spiral	1.303	1808	10.264	0.945	0.545	Kelly et al
25	Falcon concentrator	8.638	1024	7.534	0.287	0.042	Honaker et al
26	Spiral Concentrator	2.040	1677	12.524	0.727	0.117	Nicol and Bensley
27	Spiral Concentrator	2.268	1625	13.638	0.568	0.226	Apodaca
28	Water Only Cyclone	6.993	818	3.532	0.341	0.046	Miller
29	Heavy media cyclone	4.802	1384	12.293	0.231	0.014	Deurbrouck & Hudy

* Partition number Y is expressed in fraction, particle size d is expressed in millimeters and particle density ρ is expressed in kg/ m^3.

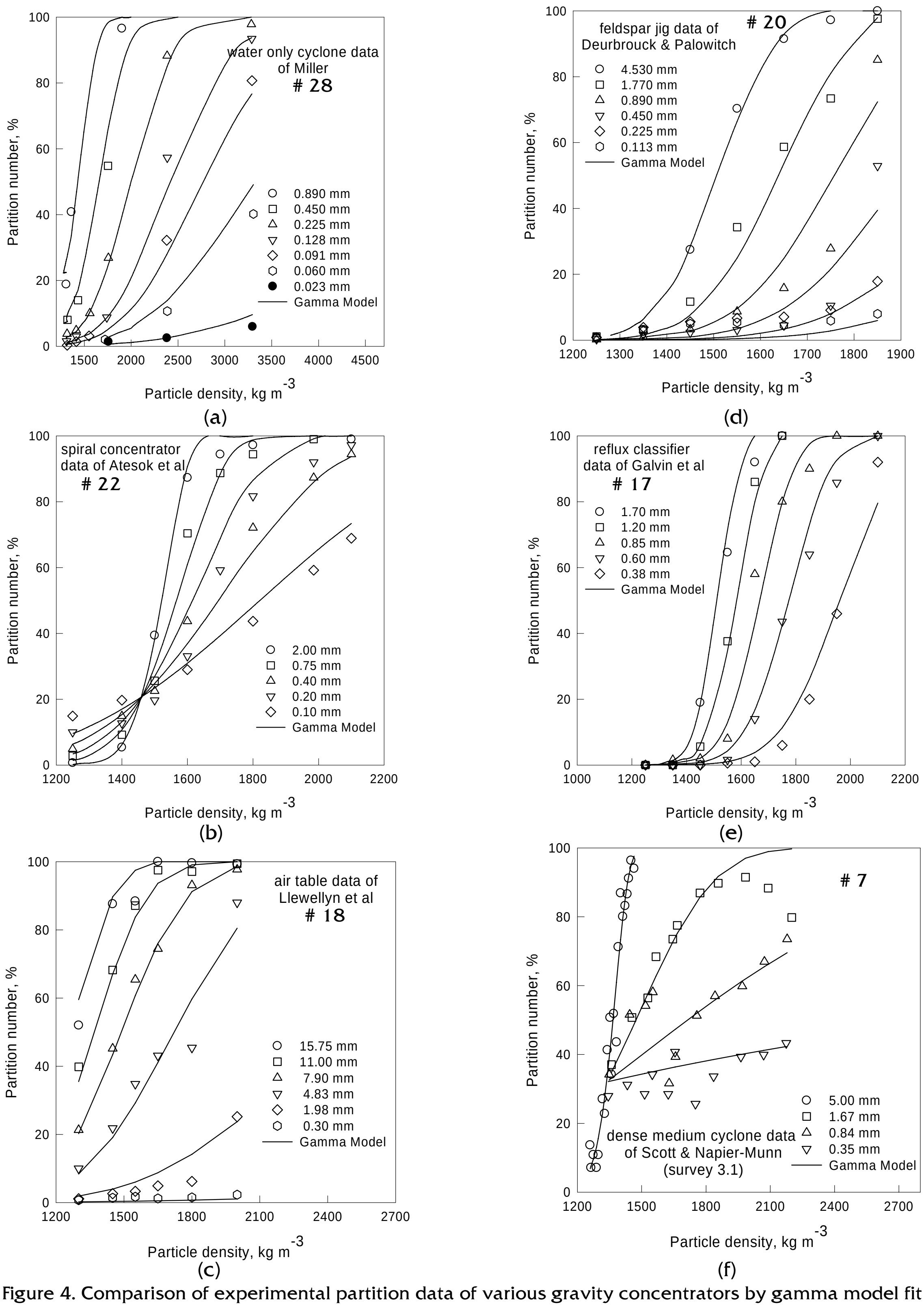

Figure 4. Comparison of experimental partition data of various gravity concentrators by gamma model fit (The number in each frame refers to the serial number in Table 1)

5. DISCUSSION

Separation indices such as cut density ρ_{50} and Ecart probable Ep, which are functions of particle size, are normally exercised to assess the performance of gravity concentrators. This section discusses the derivation of the expressions to compute these separation indices.

Computation of cut density ρ_{50} and Ecart probable Ep from gamma model requires establishment of functional relations between the internal parameters of the model, namely a and z for 25%, 50% and 75% partition numbers. In the absence of any analytical expressions for inversing the gamma function at these partition numbers, we resort to developing regression equations that relate a and z for 25%, 50% and 75% partition numbers and to use these approximate relationships to develop Eqs. (12) and (13) for cut density and Ecart probable.

For the normally encountered a values within the range of 0.18 to 15, the relations between a and z at 25%, 50% and 75% partition numbers are approximated by Eqs. (9), (10) and (11) respectively. The fit of these equations with actual data are graphically represented in Figure 5.

$$\ln(z) = \left(\frac{1.735[\ln(a)] - 1.247}{1 + 0.231[\ln(a)] - 0.037[\ln(a)]^2} \right) \tag{9}$$

$$\ln(z) = \left(\frac{1.456[\ln(a)] - 0.387}{1 + 0.169[\ln(a)] - 0.019[\ln(a)]^2} \right) \tag{10}$$

$$\ln(z) = \left(\frac{1.032[\ln(a)] + 0.335}{1 + 0.104[\ln(a)] - 0.028[\ln(a)]^2} \right) \tag{11}$$

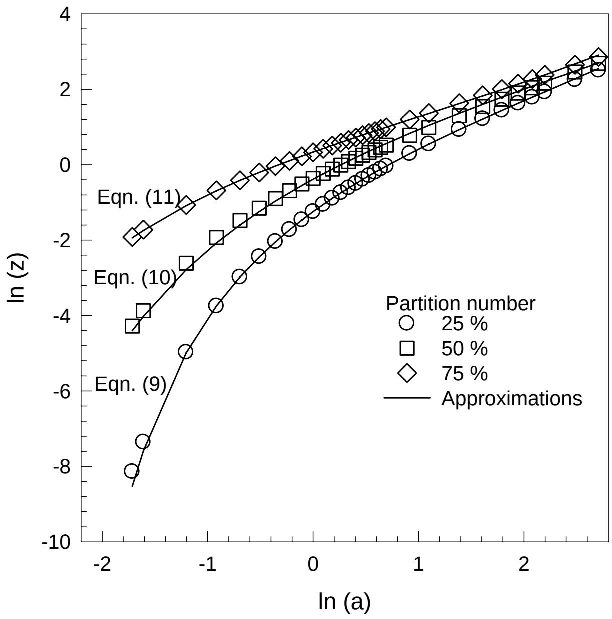

Figure 5. Comparison of approximate relations between the model parameters a and z for 25%, 50% and 75% partition numbers with the actual values from gamma inversion

The expressions for cut size and Ecart probable are thus given by

$$\rho_{50} = \rho_p \left(\exp\left[\frac{1.456[\ln(a)] - 0.387}{1 + 0.169[\ln(a)] - 0.019[\ln(a)]^2} \right] \right)^{\left(\frac{1}{ud^v} \right)} \tag{12}$$

The expressions for cut size and Ecart probable are thus given by

$$\rho_{50} = \rho_p \left(\exp\left[\frac{1.456[\ln(a)] - 0.387}{1 + 0.169[\ln(a)] - 0.019[\ln(a)]^2} \right] \right)^{\left(\frac{1}{ud^v}\right)} \tag{12}$$

$$Ep = \frac{\rho_p}{2} \left(\left(\exp\left[\frac{1.032[\ln(a)] + 0.335}{1 + 0.104[\ln(a)] - 0.028[\ln(a)]^2} \right] \right)^{\left(\frac{1}{ud^v}\right)} - \left(\exp\left[\frac{1.735[\ln(a)] - 1.247}{1 + 0.231[\ln(a)] - 0.037[\ln(a)]^2} \right] \right)^{\left(\frac{1}{ud^v}\right)} \right) \tag{13}$$

Equations (12) and (13) suggest that the evaluation of separation indices from the gamma model requires knowledge of all the model parameters a, ρ_p, u and v. Figure 6 shows the validity of Eqs. (12) and (13) for particle separation in a Falcon concentrator using data of Honaker *et al.* (2000). The representations for the separation indices are similar to other sets of data not shown here. Simulation studies indicate higher values of parameter u improve sharpness of separation. Accordingly it is argued that the parameter u is directly proportional to centrifugal accelerating force acting on the particles and inversely proportional to viscosity of the medium.

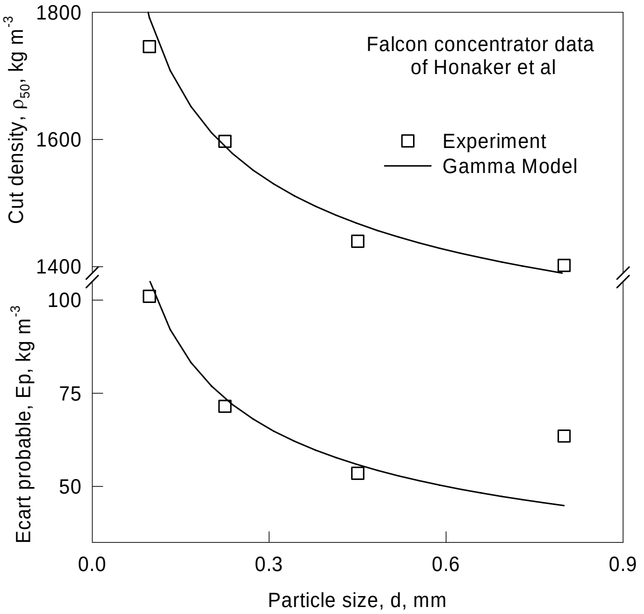

Figure 6. Comparison of actual cut density (ρ_{50}) and Ecart probable (Ep) values with equations (12) and (13) for a Falcon concentrator (Data from Honaker *et al.*, 2000)

6. CONCLUSIONS

This paper describes an empirical regularized gamma function to represent the partition surface of gravity concentrators. The model derivation is rooted in the pivot phenomena observed with all kinds of gravity concentrators. The ability of the model to describe the bivariate effects of particle size and particle density on partition coefficients has been examined with many sets of measured data taken from literature. In spite of differences in equipment geometry and particulate flow profiles within the separator, the model adequately describes partition data of all kinds of gravity concentrators. The model can thus be employed for monitoring the separation efficiency or for on line control of gravity concentrators by relating model parameters with design and operational variables that influence particle separation. Equations for computing separation indices such as cut density and Ecart probable are proposed.

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NOMENCLATURE

A parameter in Eq. (4) that incorporates viscous and particle accelerating forces
a parameter in Eq. (7) that determines pivot partition number
B parameter in Eq. (4) that reveals strength of drifting fluid
c parameter in Eq. (4) that represents flow conditions of the separator
d particle size (in mm)
Ep cart probable (in kg/m^3)
k parameter in Eq. (3) that captures viscosity effects
m sharpness index of Plitt model
n parameter in Eq. (3) that represents degree of turbulance in the separator
u parameter in Eq. (7) that captures centrifugal and viscosity effects
v parameter in Eq. (7) that represents degree of turbulance
x ratio of particle density to cut density
Y partition number, a function of particle size and particle density
Y_p pivot partition number, representing fraction of by-pass in gravity concentrators
z a function of particle size and particle density, defined in Eq. (6)

Greek symbols

α sharpness index of Lynch and Rao model
ρ particle density (in kg/m^3)
ρ_p pivot density (in kg/m^3)
ρ_{50} cut density (in kg/m^3)
γ, Γ gamma functions in Eq. (7)