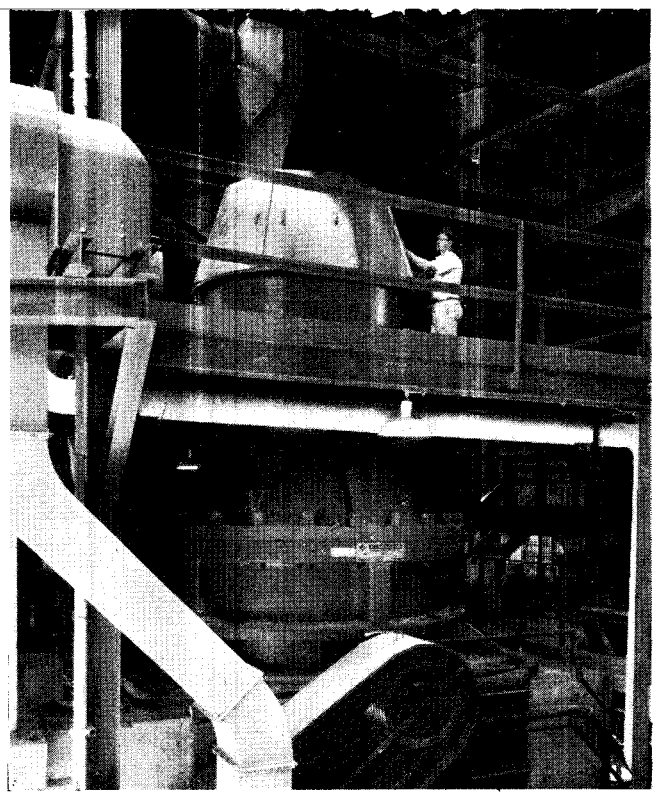


Fig. 1. 84-in. Hydrocone crusher for tertiary crushing of Semi-Taconite ore (specular hematite). (Humboldt Mining Co., Humboldt, Michigan.)



CRUSHING & GRINDING CALCULATIONS PART I

The crushing and grinding of ores, rocks and minerals is an industrial process of great importance. Specialized engineering knowledge is required for the solution of practical problems in particle size reduction, and codification of this knowledge has hardly begun. The present paper is an attempt to assemble a highly condensed summary of the principal calculation methods which the author has found useful. References are given to articles with a more extensive explanation and examples of calculations.

by FRED C. BOND

COMMUNITION theory is concerned with the relationship between energy input and the product particle size made from a given feed size. It continues to be a rich field of controversy.

The oldest theory (1867) is that of RITTINGER,¹ and it still has adherents. He stated that the area of the new surface produced by crushing or grinding is directly proportional to the useful work input. The surface area of a ton of particles of uniform diameter d is proportional to $1/d$, and according to RITTINGER the useful work input per ton is also proportional to $1/d$. However, the measured surface energy of the new surface area produced is only a very small fraction of the order of $1/1000$ of the energy input actually required to produce that surface in commercial crushing and grinding. Nearly all of the required energy input appears as heat after the particles are broken.

The second theory (1885) is that of KICK.² He stated that the work required is proportional to the reduction in volume of the particles concerned. Where f is the diameter of the feed particles and p is the diameter of the product particles, the reduction ratio Rr is f/p . According to KICK, the work input required per ton is proportional to $\log Rr/\log 2$.

Since neither theory agrees with commercial crushing and grinding results, the author developed the Third Theory in 1951.³ According to this theory, the work input is proportional to the new crack tip length produced in particle breakage, and equals the work represented by the product minus that represented by the feed. In particles of similar shape, the crack tip length is equivalent to the square root

of one-half the surface area, and the new crack length is proportional to $1/\sqrt{p} - 1/\sqrt{f}$.

For practical calculations the size in microns which 80 per cent passes is selected as the criterion of particle size. The diameter in microns which 80 per cent of the product passes is designated as P , the size which 80 per cent of the feed passes is designated as F , and the work input in kilowatt hours per short ton is W . The basic Third Theory equation is:

$$W = \frac{10 Wi}{\sqrt{P}} - \frac{10 Wi}{\sqrt{F}} \quad \dots (1)$$

where Wi is the work index. The work index is the comminution parameter which expresses the resistance of the material to crushing and grinding. Numerically the work index is the kWh per short ton required to reduce the material from theoretically infinite feed size to 80 per cent passing 100 microns, equivalent to about 67 per cent passing 200 mesh. When any three of the quantities in Equation (1) are known, the fourth can be found by transposing the equation. Useful forms are shown in (1a) and (1b) below:

$$Wi = W / \left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}} \right) \quad \dots (1a)$$

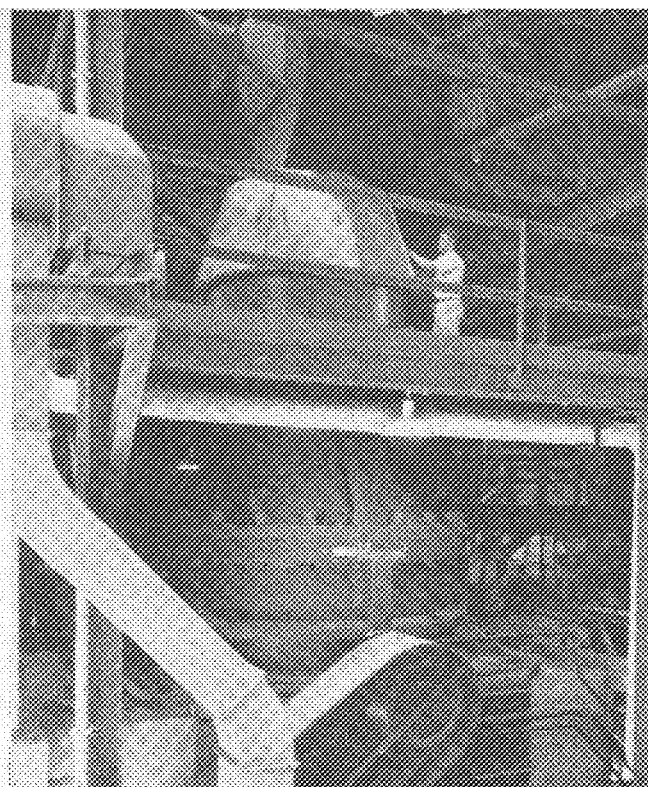
$$P = \left(\frac{10 Wi \sqrt{F}}{W \sqrt{F} + 10 Wi} \right)^2 \quad \dots (1b)$$

The work input in joules or watt-seconds per gram equals $3.97 W$.

If the material is homogeneous to size reduction, its Wi value will continue constant for all size reduction stages.

FRED C. BOND is the Senior Staff Engineer in the Process Machinery Department, Allis-Chalmers Manufacturing Co., Milwaukee, U.S.A. He is a member of A.I.M.E. and served as chairman of the Committee on Comminution Research from 1948 to 1951.

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Fig. 2. Exponential size distribution plot of ore with natural grain size between 100 and 150 mesh. Exposure ratio $E_r = 0.30$, $P = 80$ per cent passing 400 microns. Crack length equals $Cr = 24.4$ cm/cc of solid.

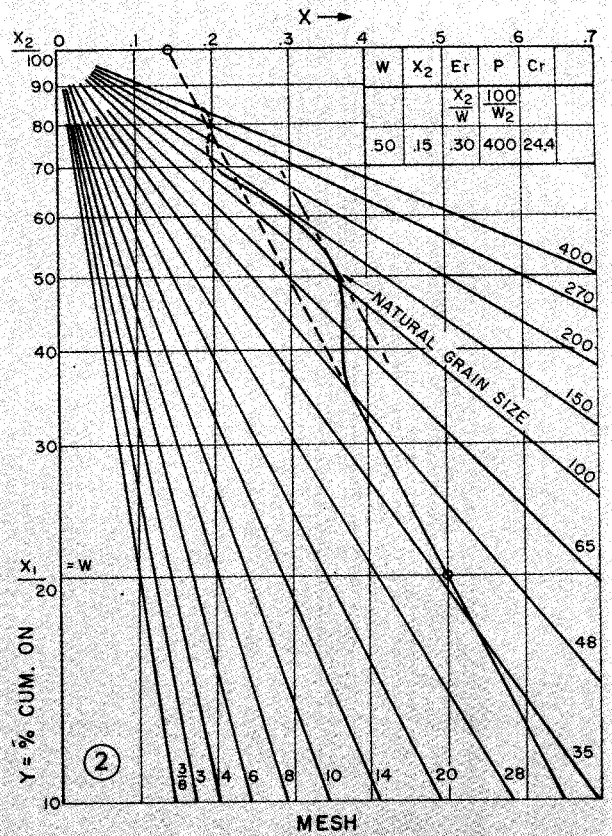
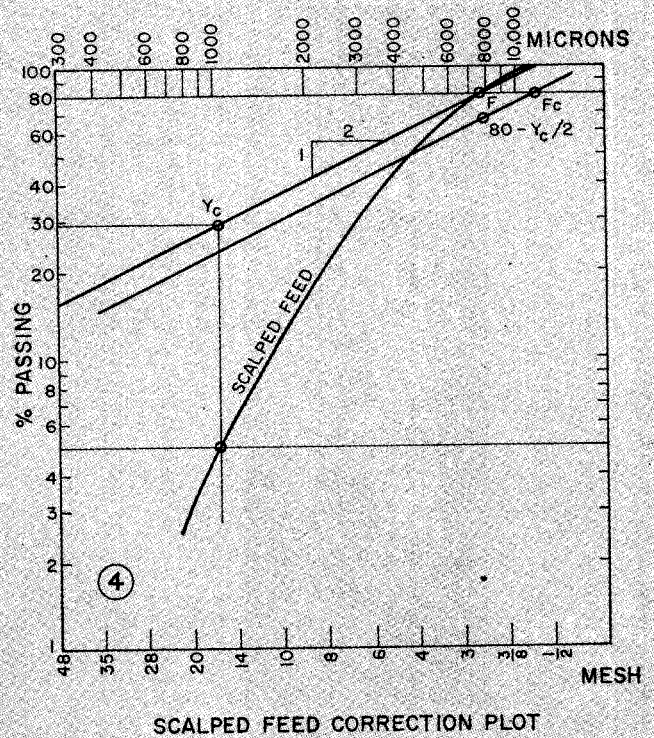
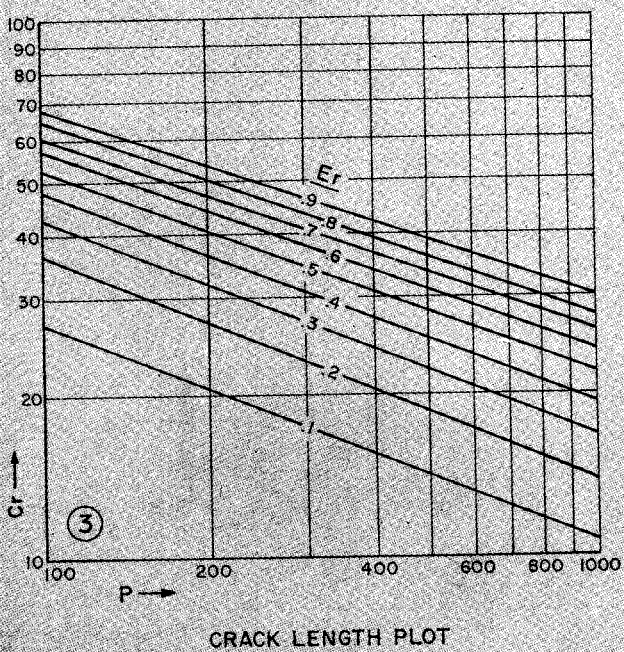


Fig. 3. Crack length plot from Table I—from 80 per cent passing 100 microns to 80 per cent passing 1000 microns for all values of exposure ratio E_r .

Fig. 4. Scalped feed correction plot. Scalped feed with 80 per cent passing 7900 microns ($F = 7900$), $Y = 29$ per cent, $80 - Y/2 = 65.5$ per cent. Slope = 1:2. Corrected feed size $F_c = 12,000$ microns.



However, heterogeneous structures in rock are common. For instance, certain materials have a natural grain size, and their Wi values will be larger below that size than above it. A loosely cemented sandstone of 48-mesh silica grains will have a larger Wi for a product with broken particles finer than 48 mesh than for a coarser product.

The efficiency of the reduction machine may also influence the operating work index. For instance, a ball mill grinding an ore from 80 per cent—14 mesh to 80 per cent—100 mesh will have a lower operating Wi value with 1.5-in. grinding balls than with oversize 3-in. balls.

A material may have an *induced* grain size resulting from some preferential sizing action which changes its natural size distribution. Undersize grinding balls can have this effect.

Laboratory determinations of the work index show the resistance to breakage at the size range tested, and any variation in the Wi values in tests at different product sizes shows that the material is not homogeneous to size reduction. For this reason laboratory tests should preferably be made at or near the product size required in commercial grinding.

The operating work index from transposed Equation (1a) can be calculated from size reduction in commercial plants to compare the plant efficiency with laboratory test results, to compare efficiencies of the different plant size reduction stages, or to compare the efficiencies of different plants treating similar materials. The work index is particularly valuable in predicting the size and capacity of new installations. Table IIIA in the appendix lists the average work index values of 82 different materials.

Three Principles

Comminution phenomena have recently been redacted into three principles,⁴ which are useful guides for the consideration of all crushing and grinding data.

The *First Principle* states that, since energy input is necessary for size reduction, all feed particles of finite size have a certain energy register, or energy level, which must be added to the energy input during crushing or grinding to obtain the energy register of the product. All statements of the energy utilized in comminution must satisfy this condition:

$$\text{energy input} = \text{energy register of product} - \text{energy register of feed.}$$

The Third Theory work index Equation (1) follows this principle, with the energy register equal to the total specific energy input in kWh per short ton. The work index Wi is the energy register to 80 per cent passing 100 microns.

If the energy which has been expended in preparing the feed particles is neglected in analysing comminution data, the first principle has been violated, and application of the calculated result to different feed and product sizes will be distorted.

The *Second Principle* states that the useful work input in crushing and grinding is proportional to the length of the new crack tips produced. In ordinary crushing and grinding, rock particles absorb strain energy and are deformed under compression or shear until the weakest flaw in the particle fails with the formation of a crack tip. This minute change of shape causes other crack tips to form at other weak flaws, and the particle breaks, releasing the bulk of the strain energy as heat. The strain energy required to break is proportional to the length of the crack tips formed, since the additional energy required to extend the crack tips to breakage is supplied by the flow of the surrounding resident strain energy to the crack tips.

Since the crack tip length is proportional to the square root of the new surface area produced, the specific work input required is inversely proportional to the square root of the product particle diameter minus that of the feed

diameter, as shown in the work index Equation (1). Crushing and grinding machines are essentially devices for the conversion of mechanical energy into strain energy into heat, under conditions which promote material breakage.

The energy register as used herein represents the specific energy which has passed through the material as strain energy, and includes heat losses and losses due to friction and other causes. It does not correspond to the energy content of the material.

The *Third Principle* deals with the relationship of particle flaws to material breakage. A flaw is defined as any structural weakness in a particle which may develop into a crack tip under strain. Flaws are always present in brittle materials and may cause wide variations in the breaking strengths of apparently similar particles.

The weakest flaw in a particle determines its breaking strength in crushing and grinding. It also controls the number of particles produced by breakage. Particles with the weakest flaws break most easily and produce the largest product particles. However, they are not necessarily easier to grind to a given product size requiring several stages of breakage than are particles of the same size whose weakest flaw is stronger.

The Third Principle states that the weakest flaw in a particle determines its breaking strength but not its work index. The work index is controlled by the average flaw structure throughout the entire size range tested. Work index variations at different product sizes result from flaw concentrations or shortages at those sizes, usually caused by natural grain sizes.

Evaluation of Particle Size Distribution

The usual standard screen scale consists of a series of sieves with square openings differing by $\sqrt{2}$, based upon the 200-mesh sieve opening of 74.2 microns. There are 25,400 microns in an inch. A screen analysis size distribution of a crushed or ground product consists of a listing of the per cent weight passing or retained on each sieve in the series.

There is probably a definite law which governs the regular size distribution of crushed or ground products; however, none of the proposed laws has been generally accepted as yet.

Size distribution analyses of crushed and ground products are commonly plotted on log-log paper with (y) the per cent passing as ordinate and the particle diameter (x) in microns as abscissa. Such plots of complete samples usually show a fairly straight line for the finer particle size range which begins to curve in the coarser sizes and often approaches tangency with the 100 per cent passing line at the top of the plot. The size 80 per cent passes may be found from the curved portion of the plot for use in the work index Equation (1).

When the straight lower portion of the plotted line is extended at its slope α , it intercepts the 100 per cent passing line at k_{100} microns. It follows a *power* law defined by the GATES-GAUDIN-SCHUHMAN equation,⁵ which is

$$y = 100 \left(\frac{x}{k_{100}} \right)^\alpha \quad \dots (2)$$

From this equation the surface area Sc in cm^2 per gram of cubical particles of density ρ , with 100 per cent passing k_{100} microns and slope α to a grind limit of Li microns is:

$$Sc = \frac{60,000 \alpha}{\rho k_{100}(1-\alpha)} \left[\left(\frac{k_{100}}{Li} \right)^{1-\alpha} - 1 \right] \quad \dots (3)$$

The grind limit Li has recently been assigned the value of 0.1 micron,⁶ equivalent to 1000 Ångstrom units. This is about 200 times the unit space lattice of quartz and other rock forming minerals.

The slope α is often about 0.5, but may approach unity.

Crushing or grinding in closed circuit produces less fines than open-circuit operation, and causes α to increase. Removal of fines before reduction has the same effect. As a material is ground finer, its value of α often appears to decrease.

The log-log size distribution plot is convenient. However, the usual curvature in its upper part indicates that the actual size distribution law is of the exponential type with a variable exponent, rather than of the power type with the constant exponent α .

Exponential Size Distribution Plots

A method of plotting has been developed with yields size distribution lines that are apparently quite straight for homogeneous materials.⁶ They follow the exponential equation.

$$Y = 100 - y = b/e^{AX} = b/10^{AX} \quad \dots (4)$$

$$AX = \log b - \log Y \quad \dots (4a)$$

X_1 represents w , the energy register in kWh/ton divided by the work index Wi at the 80 per cent passing base line where $Y = 20$. The per cent cumulative retained Y is $100 - y$, b is the $100 - y$ intercept, and A is the slope. On semi-log paper Y is measured on the vertical logarithmic scale, and X is on the horizontal linear scale. Diagonal straight lines are drawn radiating from the upper left-hand corner of the chart, which represent each mesh size on the $\sqrt{2}$ screen scale and cross the 80 per cent passing horizontal base line. Each diagonal line represents a mesh size testing sieve with an opening of P_1 microns, and crosses the base line at $w = 10/\sqrt{P_1}$. The diagonal lines can be assigned various mesh sizes, with the proper relationship between X_1 and w .

This plot is not as convenient as the log-log plot, but it has several advantages. The first is that the 80 per cent passing size P can be found with less error from $P = 100/w^2$, where w is the value of X_1 at the base line 20.

Another advantage is the delineation of natural or induced grain sizes. As the size distribution line proceeds up the chart approaching the finer sieve sizes, a curved loop to the right of the indicated straight line shows a grain size deficiency, culminating at the natural grain size where the loop becomes parallel to the straight line. The compensating grain size excess is shown by the return of the loop to the straight line. If laboratory determinations of the work index are made at the different sieve sizes, the low Wi values will increase as the grain deficiency sizes approach the natural grain size, and decrease at the grain excess sizes where the loop returns to the straight line.

The natural grain size in ores usually corresponds to the unlocking or mineral liberation size to which the ore must be ground before concentration. The exponential method of plotting the size distribution furnishes a very good indication of the unlocking size when the amount of the mineral to be concentrated is large. This is particularly true of iron ores, and the exponential plots show clearly the different unlocking properties of autogenous and conventional grinds.

Much additional information can be obtained from this type of size distribution plot, including crack length values.⁶ In the ball mill grindability tests at 60 joules input per mill revolution, the joules required to produce 1 cm of new crack length in material of homogeneous breakage with specific gravity Sg is approximately $Wi Sg/11$.

The exposure ratio Er is related to b in the exponential size distribution Equation (6) as follows, where $Er = X_{100}/w$:

$$\log b = \frac{2 - 1.301 Er}{1 - Er} \quad \dots (4b)$$

The data in Table I can be plotted on six sheets of single cycle log-log paper to make a set of charts from which the

crack length Cr of any regular crushed or ground product can be found graphically when its 80 per cent passing size P and exposure ratio Er are known from an exponential size distribution plot. On the first sheet the Cr values for $P = 1$ micron are plotted on the left-hand side, and the values for $P = 10$ microns are plotted on the right-hand side. Each pair of points is connected by a straight line marked with its Er value, and intermediate lines can be drawn using a logarithmic rule. The second sheet is made by plotting values for $P = 10$ on the left-hand side and $P = 100$ on the right-hand side, and so on for the set of six charts, which cover the entire operating size range.

TABLE I—Crack Length Values for Plotting Cr (cm/cc)

P microns	Cr						
	1	10	100	1000	10,000	100,000	1,000,000
Er							
0.10	207.0	79.8	27.3	10.65	3.76	1.37	0.495
0.20	212.0	93.0	36.6	14.00	5.25	1.93	0.735
0.30	208.5	102.1	42.5	16.89	6.60	2.47	0.936
0.40	202.0	110.0	48.1	19.56	7.87	2.96	1.136
0.50	191.9	115.2	52.7	21.95	9.00	3.44	1.337
0.60	183.0	119.4	57.1	24.25	10.04	3.91	1.523
0.70	176.0	123.0	60.8	26.35	11.00	4.37	1.699
0.80	165.8	125.8	64.3	28.40	11.94	4.81	1.860
0.90	160.0	128.1	67.5	30.35	12.85	5.22	2.029

Surface Area Calculations of Ground Products

Approximate surface areas in sq. cm per gram of equivalent cubes for log-log size distribution plots can be calculated from Equation (3), using a grind limit Li of 0.10 micron.

When the crack length Cr has been found,⁶ the surface of equivalent cubes in sq. cm per cc of solids is $2 Cr^2$.

The WAGNER surface area Sw in sq. cm/gram is approximately equal to the BLAINE air permeability surface to the power 0.92, or $Sw = (B1)^{0.92}$.

The 80 per cent passing size P in microns has the following approximate relationship to the BLAINE and WAGNER surface areas:

$$P = \left(\frac{20,300}{B1} \right)^2 = \frac{3.63 \times 10^8}{(Sw)^{2.15}} \quad \dots (5)$$

$$\text{and } \log P = 2 \log (20,300/B1) = 8.56 - 2.15 \log (Sw) \quad \dots (5a)$$

Work Index from Laboratory Tests

Equations have been derived for finding the work index Wi from several types of laboratory crushability and grindability tests,⁷ as described below.

Crushability Test

Pieces of broken rock passing a 3-in. square and retained on a 2-in. square are mounted between two opposing equal 30-lb weights which swing on wheels. When the wheels are released the weights strike simultaneously on opposite sides of the measured smallest dimension of the rock. The height of fall is successively increased until the rock breaks. The impact crushing strength in foot-pounds per inch of rock thickness is designated as C , and Sg is the specific gravity. The work index is found from the average of 10 breaks, where

$$Wi = 2.59 C/Sg \quad \dots (6)$$

Rod Mill Grindability Test

The feed is crushed to $-\frac{1}{2}$ in., and 1250 cc packed in a graduated cylinder are weighed, screen analysed, and ground dry in closed circuit with 100 per cent circulating load in a 12 in. dia. by 24 in. long tilting rod mill with a wave-type lining and revolution counter, running at 46 rpm. The grinding charge consists of six 1.25 in. dia. and two 1.75 in. dia. steel rods 21 in. long and weighing 33,380 grams.

In order to equalize segregation at the mill ends, it is rotated level for eight revolutions, then tilted up 5° for one revolution, down 5° for another revolution, and returned

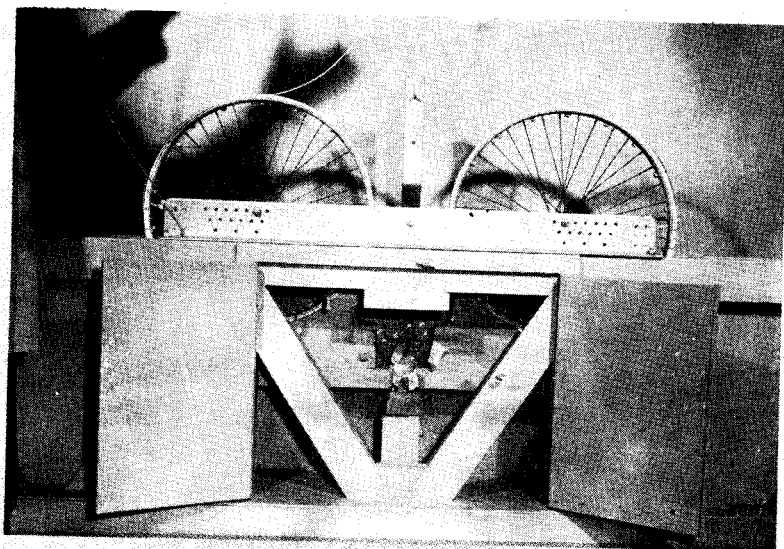


Fig. 5. Twin pendulums impact crushing device for measuring work index. Picture of specimen breakage taken at instant of impact.

to level for eight revolutions continuously throughout each grinding period.

Tests are made at all mesh sizes from 4 to 65 mesh. At the end of each grinding period the mill is discharged by tilting downward at 45° for 30 revolutions, and the product is screened on sieves of the mesh size tested. The sieve undersize is weighed, and fresh unsegregated feed is added to the oversize to make its total weight equal to that of the 1250 cc originally charged into the mill. This is returned to the mill and ground for the number of revolutions calculated to give a circulating load equal to the weight of the new feed added. The grinding period cycles are continued until the net grams of sieve undersize produced per revolution reaches equilibrium and reverses its direction of increase or decrease. Then the undersize product and circulating load are screen analysed, and the average of the last three net grams per revolution (*Grp*) is the rod mill grindability.

Where *F* is the size in microns which 80 per cent of the new rod mill feed passes, and *P*₁ is the opening of the sieve size tested in microns, then the rod mill work index *Wi* is calculated from the following revised (1960) equation:

$$Wi = 62/(P_1)^{0.23} \times (Grp)^{0.625} \times \left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}} \right) \dots (7)$$

This *Wi* value should conform with the motor output power to an average overflow rod mill of 8 ft interior diameter grinding wet in open circuit. For dry grinding the work input should be multiplied by 1.30. Where *D* is the mill diameter inside the lining in feet, the work input should be multiplied by $(8/D)^{0.20}$.

Ball Mill Grindability Test

The standard feed is prepared by stage crushing to all passing a 6 mesh sieve, but finer feed can be used when necessary. It is screen analysed and packed by shaking in a 1000-cc graduated cylinder, and the weight of 700 cc is placed in the mill and ground dry at 250 per cent circulating load. The mill is 12 in. × 12 in. with rounded corners, and a smooth lining except for a 4 in. × 8 in. hand hole door for charging. It has a revolution counter and runs at 70 rpm. The grinding charge consists of 285 iron balls weighing 20,125 grams. It consists of about 43 1.45-in. balls, 67 1.17-

in. balls, 10 1-in. balls, 71 0.75-in. balls, and 94 0.61-in. balls with a calculated surface area of 842 sq. in.

Tests are made at all sieve sizes below 28 mesh. After the first grinding period of 100 revolutions, the mill is dumped, the ball charge is screened out, and the 700 cc of material is screened on sieves of the mesh size tested, with coarser protecting sieves if necessary. The undersize is weighed, and fresh unsegregated feed is added to the oversize to bring its weight back to that of the original charge. Then it is returned on to the balls in the mill and ground for the number of revolutions calculated to produce a 250 per cent circulating load, dumped and rescreened. The number of revolutions required is calculated from the results of the previous period to produce sieve undersize equal to 1/3.5 of the total charge in the mill.

The grinding period cycles are continued until the net grams of sieve undersize produced per mill revolution reaches equilibrium and reverses its direction of increase or decrease. Then the undersize product and circulating load are screen analysed, and the average of the last three net grams per revolution (*Gbp*) is the ball mill grindability.

When *F* is the size in microns which 80 per cent of the new ball mill feed passes, *P* is the microns which 80 per cent of the last cycle sieve undersize product passes, and *P*₁ is the opening in microns of the sieve size tested, then the ball mill work index *Wi* is calculated from the following revised (1960) equation:

$$Wi = 44.5/(P_1)^{0.23} \times (Gbp)^{0.82} \left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}} \right) \dots (8)$$

The average value of *P* at 100 mesh is 114 microns, at 150 mesh it is 76 microns, at 200 mesh it is 50, and at 325 mesh it is 26.7. These values of *P* are to be used in Equation (8) when *P* cannot be found from size distribution analyses.

The *Wi* value from Equation (8) should conform with the motor output power to an average overflow ball mill of 8 ft interior diameter grinding wet in closed circuit. For dry grinding the work input should normally be multiplied by 1.30. However, ball coating and packing can increase the work input in dry grinding.

Where *D* is the mill diameter inside the lining in ft, the work input should be multiplied by $(8/D)^{0.20}$.

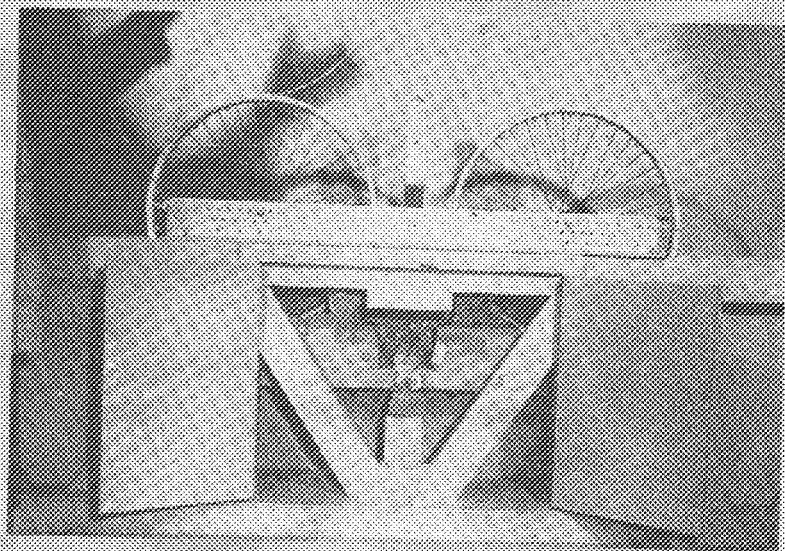


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to level for eight revolutions continuously throughout each grinding period.

Tests are made at all mesh sizes from 4 to 65 mesh. At the end of each grinding period the mill is discharged by tilting downward at 45° for 30 revolutions, and the product is screened on sieves of the mesh size tested. The sieve undersize is weighed, and fresh unsegregated feed is added to the oversize to make its total weight equal to that of the 1250 cc originally charged into the mill. This is returned to the mill and ground for the number of revolutions calculated to give a circulating load equal to the weight of the new feed added. The grinding period cycles are continued until the net grams of sieve undersize produced per revolution reaches equilibrium and reverses its direction of increase or decrease. Then the undersize product and circulating load are screen analysed, and the average of the last three net grams per revolution (*Grp*) is the rod mill grindability.

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$$W_1 = 62(P_1)^{0.23} \times (Grp)^{0.425} \times \left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}} \right) \dots (7)$$

This *W*₁ value should conform with the motor output power to an average overflow rod mill of 8 ft interior diameter grinding wet in open circuit. For dry grinding the work input should be multiplied by 1.30. Where *D* is the mill diameter inside the lining in feet, the work input should be multiplied by (8/*D*)^{0.20}.

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The standard feed is prepared by stage crushing to all passing a 6 mesh sieve, but finer feed can be used when necessary. It is screen analysed and packed by shaking in a 1000-cc graduated cylinder, and the weight of 700 cc is placed in the mill and ground dry at 250 per cent circulating load. The mill is 12 in. × 12 in. with rounded corners, and a smooth lining except for a 4 in. × 8 in. hand hole door for charging. It has a revolution counter and runs at 70 rpm. The grinding charge consists of 285 iron balls weighing 20.125 grams. It consists of about 43 1.45-in. balls, 67 1.17-

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Tests are made at all sieve sizes below 28 mesh. After the first grinding period of 100 revolutions, the mill is dumped, the ball charge is screened out, and the 700 cc of material is screened on sieves of the mesh size tested, with coarser protecting sieves if necessary. The undersize is weighed, and fresh unsegregated feed is added to the oversize to bring its weight back to that of the original charge. Then it is returned on to the balls in the mill and ground for the number of revolutions calculated to produce a 250 per cent circulating load, dumped and rescreened. The number of revolutions required is calculated from the results of the previous period to produce sieve undersize equal to 1/3.5 of the total charge in the mill.

The grinding period cycles are continued until the net grams of sieve undersize produced per mill revolution reaches equilibrium and reverses its direction of increase or decrease. Then the undersize product and circulating load are screen analysed, and the average of the last three net grams per revolution (*Gbp*) is the ball mill grindability.

When *F* is the size in microns which 80 per cent of the new ball mill feed passes, *P* is the microns which 80 per cent of the last cycle sieve undersize product passes, and *P*₁ is the opening in microns of the sieve size tested, then the ball mill work index *W*₁ is calculated from the following revised (1960) equation:

$$W_1 = 44.5(P_1)^{0.23} \times (Gbp)^{0.425} \times \left(\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}} \right) \dots (8)$$

The average value of *P* at 100 mesh is 114 microns, at 150 mesh it is 76 microns, at 200 mesh it is 50, and at 325 mesh it is 26.7. These values of *P* are to be used in Equation (8) when *P* cannot be found from size distribution analyses.

The *W*₁ value from Equation (8) should conform with the motor output power to an average overflow ball mill of 8 ft interior diameter grinding wet in closed circuit. For dry grinding the work input should normally be multiplied by 1.30. However, ball coating and packing can increase the work input in dry grinding.

Where *D* is the mill diameter inside the lining in ft, the work input should be multiplied by (8/*D*)^{0.20}.

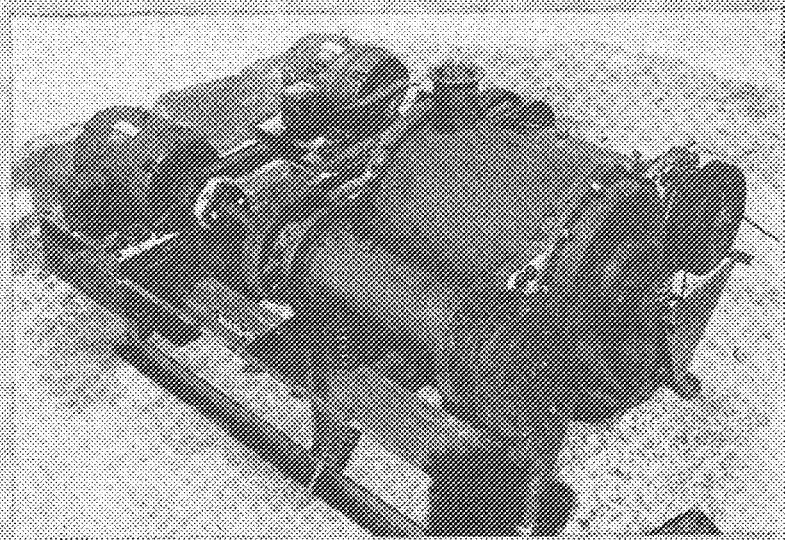


Fig. 6. 30-in. vibrating ball mill.

Hardgrove Grindability Rating

Where Hd represents the Hardgrove grindability rating,¹³ the equivalent wet grinding work index is found from:

$$W_i = 435/(Hd)^{0.61} \quad \dots (9)$$

Crushing

Crushing is accomplished by contact with metal or other surfaces maintained in a fixed position or in a rigidly constrained motion path, although many crushers have safety features which allow release under excessive pressure. This is contrasted with grinding, which is accomplished by the free motion in response to gravity and other forces of unconnected media such as rods, balls, rock pieces and pebbles.

Free media grinding has several inherent advantages over fixed media crushing, and as reduction machinery increases in size and strength larger particles become amenable to grinding which could formerly be reduced only by crushing. Cases in point are the development of large peripheral discharge rod mills and autogenous grinding mills. However, the commercial production of particles larger than about $\frac{1}{2}$ in. is still a crushing process.

Crushing is usually done dry in several stages with small reduction ratios ranging from 3 to 6 in each stage. The machines used include: gyratory crushers, jaw crushers (both single and double toggle), crushing rolls, and impact crushers, hammer mills or pulverizers. It is done with both screened and natural feeds, in stages with screens between each stage to remove undersize, as well as in open circuit and in closed circuit with screens.

Excessive moisture, fines, or both, in the feed can cause packing in the crusher, resulting in a decrease in capacity, increase in power drawn, and increase in the crushing plant work index. This is usually remedied by screening out more fines ahead of the crusher.

Crusher motor sizes are usually limited to protect the crushers against breakage. For the same reason uncrushable pieces of metal are usually removed from the feed magnetically, or the crusher is designed to open up and let them pass through.

Crusher Product Sizes

The crusher product size which 80 per cent passes at full

capacity can be estimated from the crusher setting, eccentric throw and work index of the material.

The product sizes of jaw crushers and primary gyratory crushers with steep crushing cones are controlled principally by the open side setting of the crusher. Where O_{ss} is the open side setting of the crusher in inches at the bottom of the crushing chamber, the 80 per cent passing size P of the crusher product in microns is calculated from Equation (10). 1 in. equals 25,400 microns.

$$P = (25,400) (O_{ss}) (0.04W_i + 0.40) \quad \dots (10)$$

The product sizes of cone crushers, with their flat crushing cones and relatively high speeds, are controlled principally by the close side setting. Where C_{ss} is the close side setting of the cone crusher in inches at the bottom of the crushing chamber, as commonly determined by passing a piece of lead through the crusher, and E_{cc} is the eccentric throw in inches at the bottom of the crushing cone, the product size P is found from

$$P = \frac{(25,400) (C_{ss}) (E_{cc}) (0.02W_i + 0.70)}{(E_{cc} - 2C_{ss})} \quad \dots (11)$$

If the material is very slabby, the value of P may be somewhat larger than that indicated by Equations (10) and (11). These equations are useful when screen analyses of the crusher products are not available.

Scalped Feed to Crushers

The Third Theory equations require a "natural" feed containing the natural fines produced in the previous reduction stages. When fines are removed from the feed, the relationship between F and P is altered. In most crushing installations where fines smaller than the crusher discharge opening are removed from the feed by screening, the work input per ton of original feed is not materially decreased, except as the removal of fines prevents the abnormal condition of packing in the crusher. It has been found satisfactory to disregard the scalping operation, and to consider the feed to the screen or grizzly as equivalent feed to the crusher. This is preferable in most cases where the grizzly separating size and hourly tonnage through are not known accurately.

However, in some instances where much of the fines have been removed the correction for scalped feed must be

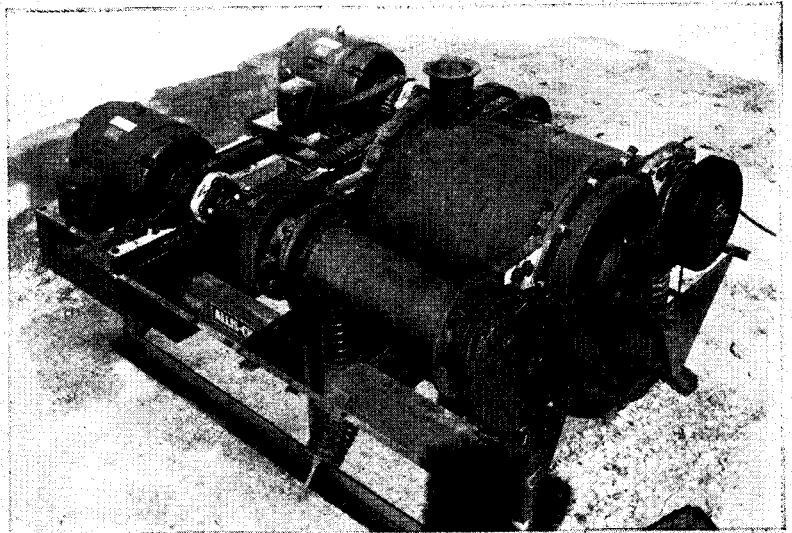


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Free media grinding has several inherent advantages over fixed media crushing, and as reduction machinery increases in size and strength larger particles become amenable to grinding which could formerly be reduced only by crushing. Cases in point are the development of large peripheral discharge rod mills and autogenous grinding mills. However, the commercial production of particles larger than about $\frac{1}{4}$ in. is still a crushing process.

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The product sizes of cone crushers, with their flat crushing cones and relatively high speeds, are controlled principally by the close side setting. Where Css is the close side setting of the cone crusher in inches at the bottom of the crushing chamber, as commonly determined by passing a piece of lead through the crusher, and Ecc is the eccentric throw in inches at the bottom of the crushing cone, the product size P is found from

$$P = \frac{(25,400) (Css) (7Ecc) (0.02Wi + 0.70)}{(7Ecc - 2Css)} \quad \dots (11)$$

If the material is very slabby, the value of P may be somewhat larger than that indicated by Equations (10) and (11). These equations are useful when screen analyses of the crusher products are not available.

Scalped Feed to Crushers

The Third Theory equations require a "natural" feed containing the natural fines produced in the previous reduction stages. When fines are removed from the feed, the relationship between F and P is altered. In most crushing installations where fines smaller than the crusher discharge opening are removed from the feed by screening, the work input per ton of original feed is not materially decreased, except as the removal of fines prevents the abnormal condition of packing in the crusher. It has been found satisfactory to disregard the scalping operation, and to consider the feed to the screen or grizzly as equivalent feed to the crusher. This is preferable in most cases where the grizzly separating size and hourly tonnage through are not known accurately.

However, in some instances where much of the fines have been removed the correction for scalped feed must be

made. This is done empirically by using that increased normal feed size F_c which is equivalent in work input per ton to the 80 per cent passing size F of the scalped feed. The per cent passing size distribution line of the scalped feed is plotted on log-log paper. A line with the normal slope of 1:2 is drawn through the 80 per cent passing point F to its intersection Y_c with the size which 5 per cent of the scalped feed passes; a parallel line is drawn through the point with co-ordinates $F, (80 - Y_c/2)$, and its intersection with the 80 per cent passing line gives the value of F_c .

When pieces all of one diameter of d microns are fed to a crusher the equivalent 80 per cent passing size F_c is that of a Third Theory size distribution line with an exposure ratio Er of 0.05 and the same crack length as the particles fed.⁶ The crack length in cm/cc is $Cr = 173/\sqrt{d}$. The F_c values are listed in Table II. When the feed consists of particles of several different diameters d without fines, the equivalent corrected feed size F_c can be computed as the weighted average of the different sizes d .

TABLE II—Equivalent 80 per cent Passing Size F_c for Particles all d Microns Diameter. Cr =Crack Length in cm/cc= $173/\sqrt{d}$. As =Slope= $0.0699/\sqrt{d}$.

Size		As Slope of sieve lines	Cr cm/cc	F_c Microns equivalent ($Er=0.05$)
Sieve Inch Mesh	Microns d ($=P_1$)			
3 in.	76,200	19.30	0.626	255,000
2 in.	50,800	15.73	0.769	163,000
1 in.	26,670	11.42	1.057	76,000
1/2 in.	18,850	9.60	1.260	50,400
3/8 in.	13,330	8.56	1.500	34,000
1/4 in.	9,423	6.79	1.782	23,000
3/16 in.	6,680	5.70	2.12	15,400
1/8 in.	4,760	4.82	2.51	10,400
3/32 in.	3,360	4.05	2.98	8,000
1/16 in.	2,380	3.14	3.55	5,500
3/64 in.	1,680	2.86	4.22	3,760
1/32 in.	1,190	2.41	5.02	2,600
3/64 in.	840	2.024	5.97	1,800
1/16 in.	590	1.699	7.13	1,220
3/32 in.	420	1.432	8.45	835
1/8 in.	297	1.205	10.01	580
3/16 in.	210	1.012	11.94	375
1/4 in.	149	.853	14.18	250
3/8 in.	105	.716	16.90	158
1/2 in.	74.2	.602	20.02	103
3/4 in.	52.5	.506	23.9	71
1 in.	44	.464	26.1	59
1 1/8 in.	37.1	.426	28.4	50
1 1/4 in.	30	.383	31.6	40
1 3/8 in.	25	.350	34.6	34
1 1/2 in.	20	.313	38.7	27.3
1 3/4 in.	15	.271	44.7	20.7
2 in.	10	.221	54.8	14.0
2 1/2 in.	5	.156	77.5	7.0

Grinding

Correction for Feed and Products

Closed-circuit grinding and complex grinding circuits which include concentration and separation equipment are best analysed by considering each circuit as an integrated unit. In closed-circuit grinding the unit consists of the mill and the classifier, with a single feed to the mill or classifier and a single classifier undersize product. Calculations from the mill discharge and circulating load are usually unsatisfactory because the harder fraction of the material accumulates, and the circulating load has an unknown higher work index than the new feed. If the closed circuit includes concentrating equipment such as magnetic separators which reject a tailing, the product of the grinding circuit is a calculated composite of the classifier undersize and the separator tailing, which should always be screen analysed for this purpose.

Calculations involving unnatural feed from which part or all of the fines have been removed should be avoided whenever calculations of integrated circuits can be substituted. However, the empirical methods described under "Scalped Feed to Crushers" can be used when necessary in grinding circuits.

When grinding tests are made in a small, continuous, pilot-plant mill of diameter D , the gross power input per

ton ground should be multiplied by $(D/8)^{0.20}$ to find the work index from Equation (1a).

Fineness Adjustment

Experience has shown that extremely fine grinding requires additional energy input beyond that indicated by the work index Equation (1). There are several possible reasons for this, including:

- The ball sizes customarily employed are too large for extremely fine particles.
- In fine grinding the closed-circuit classification is usually either inefficient or absent.
- In dry grinding of fine particles the amount of ball coating ranges from incipient to complete. Ball coating cushions the metal contacts and decreases grinding efficiency.
- In wet grinding a thick viscous pulp can cause ball cushioning and decrease grinding efficiency.
- The production of particles smaller than the grind limit Li of 0.1 micron involves breakage across the unflawed space lattice, and requires several times as much energy input as the customary breakage along planes of lattice displacement.⁶ Considerable amounts of trans-grind-limit particles may be produced in ultra-fine grinding.

When the product size P is less than 70 microns the work input W as calculated from laboratory tests is multiplied by the following empirical adjustment factor A_1 , which equals

$$\frac{P + 10.3}{1.145P}$$

The fine product factor A_1 was derived for the fine dry grinding of cement clinker, and applies to dry grinding down to P values of 15 or less. For wet fine grinding, A_1 should have a maximum value of 5.

Conversely, the plant operating work index from Equation (1a) should be divided by A_1 and by $(8/D)^{0.2}$ for direct comparison with the laboratory work index Wi .

Proper Grinding Media Sizes

The size of the grinding media is one of the principal factors affecting the efficiency and capacity of tumbling-type grinding mills. It is best determined for any particular installation by lengthy comparative plant tests with carefully kept records. However, a method of calculating the proper sizes, based upon correct theoretical principles and tested by experience, can be very helpful, particularly in new installations.

The general principle of selection should be that the proper size of the make-up grinding media is the size which will just break the largest feed particles. If the media is too large, the number of breaking contacts will be reduced, and the extreme fines made by each contact will be increased. If the media is too small, there will be wasted contacts of a force insufficient to break the particles contacted. In either case the grinding efficiency will be reduced, but the use of undersize media is usually more harmful than the use of oversize media.

- Let B = make-up ball, rod, or pebble diameter in inches;
 F = size in microns 80 per cent of the new feed passes;
 Wi = work index at the feed size F ;
 C_s = fraction of mill critical speed;
 S_g = specific gravity of material being ground;
 D = mill diameter in feet inside liners;
 K = an empirical experience constant.

For Ball Mills

In ordinary ball mill operation 1-in. steel balls will effectively grind average siliceous ore with 80 per cent passing 1 mm, or with $F = 1000$ microns or about 16 mesh. It

follows theoretically,⁸ and is confirmed by experience, that 2-in. balls are suitable for 4-mm feed, 3-in. balls for 9-mm particles, etc. The ball size should vary as the square root of the particle size to be broken.

From theoretical considerations⁸ the proper make-up size B of steel or cast iron balls is found from

$$B = \left(\frac{F}{K}\right)^{\frac{1}{2}} \left(\frac{SgWi}{100 C_s \sqrt{D}}\right)^{\frac{1}{3}} \dots (12)$$

The empirical constant K is found by experience to be 350 for wet grinding and 335 for dry grinding.

The commercial size nearest to B is ordinarily selected for the make-up ball size. However, when B is less than 1 in., it may be economical to select a larger ball size for these reasons: (1) the cost per ton of the smaller balls is increased; (2) less wear is obtained from the smaller balls before they are discharged from the mill; (3) the smaller balls may plug the grates of diaphragm discharge mills; (4)

large-diameter mills draw more power with large ball sizes.

Ball rationing, which is the regular addition of definite proportions of balls of different sizes, may be used when B is intermediate between two commercial ball sizes, or where an unusual size distribution of the feed requires the addition of some smaller balls with those of the calculated size B .

For Rod Mills

From theoretical considerations⁹ the proper diameter B of make-up steel grinding rods is found from:

$$B = \frac{F^{\frac{1}{2}}}{160} \sqrt{\frac{Wi Sg}{100 C_s \sqrt{D}}} \dots (13)$$

When the reduction ratio $Rr = F/P$ is less than 8, the calculated value of B should be increased by $\frac{1}{2}$ in.

For Pebble and Rockfed Mills

Pebbles fed to pebble mills, and the rock fed to auto-genous mills where the large pieces grind the smaller particles, are selected to have the same weight as steel balls suitable for the same service. When B is the proper make-up ball size according to Equation (12), then the proper pebble or grinding rock size of specific gravity Sg is $B \times (7.8/Sg)^{\frac{1}{3}}$.

Size Distribution of Grinding Media

All types of grinding media commonly wear down to sizes sufficiently small to discharge from the mill with the material being ground. However, in some rod mills broken and worn rods are removed manually.

It has been determined that a film of metal of unit thickness is worn from any size ball in a mill in the same grinding time. If the weight loss is periodically replaced as make-up balls of size B , the ball charge reaches an equilibrium size distribution which extends down to almost the ball size discharged from the mill. This equilibrium size distribution follows the equation

$$y = (x/B)^{3.8} \dots (14)$$

where y is the percentage of the total equilibrium charge passing any size x . Equation (14) presumably holds for grinding rods and pebbles as well as balls.

In order to obtain consistent performance in wet grinding mills, the initial media charge should be made up from the several sizes available to be similar to the equilibrium charge defined by Equation (14). This can be approximated by drawing on log-log paper the straight per cent passing line with a slope of 3.8 through 100 per cent passing size B . The initial charge composition is determined by marking the points midway between the ball or rod sizes to be used. If ball rationing is to be used, the initial charge should be proportioned between the two ball sizes fed.

In commercial rod mills no rod sizes smaller than $2\frac{1}{2}$ in. should be used in the initial charge. In commercial ball mills the minimum size used is commonly 1 in.

In dry-grinding mills the metal wear rate is so much less than in wet mills that two years or more may be required to reach equilibrium, and the initial charge can be proportioned to fit the mill feed without reference to the equilibrium charge.

The weight, volume, and surface area of steel balls or rods of diameter B inches can be found from Table III.

TABLE III
(B =Diameter in inches)

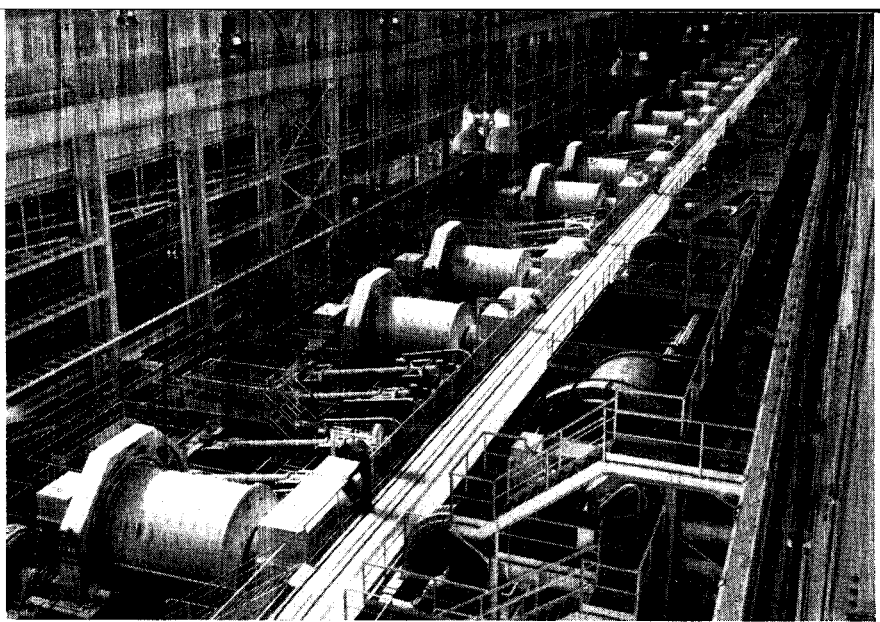
Number of balls or feet rods	lb.	cu. ft.	Sq. in. surface area
One ball	0.148 B^3	—	3.142 B^2
One ft rod	2.67 B^3	—	37.8 B^2
13,500/ B^3 balls	2,000	6.89	42,500/ B
750/ B^2 ft rods	2,000	5.13	28,400/ B
1,960/ B^3 balls	290	1	6,160/ B
146/ B^2 ft rods	390	1	5,510/ B
Equilibrium charge fed with B balls	2,000	6.89	57,500/ B

(To be continued)

Symbol*	In Equation No.	Description
A	4	Exponential plot slope.
A_1	—	Fine product adjustment factor.
A_2	26	Rod mill Rr adjustment factor.
A_3	27	Ball mill Rr adjustment factor.
B	—	Diam. in inches of balls and rods fed.
Bl	5	Blaine surface area in sq. cm/gram.
b	4	Exponential size distribution coefficient.
C	6	Ft-lb per inch, impact crushing strength.
Cl	9, 37, 38, 39, 40, 41	Per cent circulating load/100.
Cp	33, 35, 36	Microns 80 per cent of concentrate passes.
Cr	—	Total crack length—cm/cc.
Cs	12, 13, 19	Fraction of mill critical speed.
Css	11	Close side crusher setting in inches.
Cw	33	Per cent weight of concentrate in feed.
c	39, 40	Per cent passing in classifier fine product.
cpm	28, 29	Cycles per minute in free vertical oscillation.
D	19, 21-25	Inside mill diameter in feet.
Dp	27, 38	Per cent of discharge passing size P .
d	1	Microns 80 per cent passes.
Ecc	11	Eccentric throw of crusher in inches.
Er	4b	Exponential plot exposure ratio.
F	1, 2, 3, 4, 19, 32	Feed size—microns 80 per cent passes.
Fc	—	Equivalent size of scalped feed.
Fo	30, 31, 32	Maximum efficient feed size in microns.
Fp	38	Per cent of new feed passing size P .
f	5	Diameter of feed particle in microns.
fa	40	Per cent passing in mill new feeds.
Gd	33, 34	Grind differential
Gbp	8	Ball mill grindability A-C, net grams/revolution.
Grp	7	Rod mill grindability A-C, net grams/revolution.
Hd	9	Hardgrove grindability
h	28	Inches of free vertical oscillation.
K	—	Grinding media size constant.
Kvo	29	Kilowatts per ton in free vertical oscillation.
Kwb	22	Kilowatts per ton of balls.
Kwr	21	Kilowatts per ton of rods.
k_{100}	2, 3	Schuhmann microns 100 per cent passes.
L	16, 17	Length of mill interior in feet.
Li	3	Grind limit in microns.
m	39, 40	Per cent passing in mill discharge.
Nc	18	Critical mill speed in rpm.
No	20	Optimum mill speed in rpm.
Oss	10	Open side crusher setting in inches.
P	—	Product size—microns 80 per cent passes.
P_1	13, 14	Sieve opening in microns—A-C grindability.
Q	15	Feet from top of mill to charge.
Qd	23	Feet from top of mill to lowest discharge point.
ρ	3	Density.
Rf	35, 36	Recession factor between concentrate and feed.
Rp	37, 38	Per cent of oversize passing size P .
Rr	4, 5	Reduction ratio= F/P .
Rro	25	Optimum rod mill reduction ratio.
Sc	3	Surface area in sq. cm/gram.
Sg	6	Specific gravity of ore.
Ss	24	Ball slump subtraction quantity.
Sw	5	Sq cm per gram, Wagner surface.
Tb	16	Tons of balls in mill.
Tr	17	Tons of rods in mill.
t	39, 40	Per cent passing in classifier coarse product.
Vp	15	Fraction mill volume occupied.
Vpd	23	Fraction of mill volume below discharge level.
W	1, 2, 3, 4	Work input in kWh per short ton.
Wi	1, 6-13	Work index: kWh/ton to 80 per cent—100 microns.
Wt	1, 10	Work total: kWh/ton to product size.
w	—	kWh/ Wi
X	4	Exponential plot abscissa.
Y	4	Per cent cumulative retained on any size.
α	2, 3	Slope of size distribution line.
Wip	42	Work Index from plot at product size P .
Wif	42	Work Index from plot at feed size F .

*In this paper three classes of symbol are used: lower case single letters, upper case single letters and compound symbols consisting of an upper case letter followed by one or more lower case letters. They are distinguished by the sequence rule; lower case single letters are written first, followed by upper case single letters, and in every term the compound symbols are written last.

Fig. 1. Fourteen 800-hp ball mills installed at Southern Peru Copper Corp.'s concentrator, Toquepalo, Peru, grinding 30,000 tons of copper ore daily.



PART II

CRUSHING AND GRINDING CALCULATIONS

The concluding part of this article is concerned with a number of factors affecting the grinding process and the life of equipment, such as the fraction of mill volume occupied by the grinding charge, the quantities of rods and balls to be contained in a mill, wear of mill and grinding medium, and mill speed in terms of the critical speed. The effects of mill diameter, of downward slippage of ball charge, ratio oversize:feed, upon performance and power consumption are discussed. Open-circuit multiplication factors are given for converting closed-circuit work values to the open-circuit values and, finally, correction factors are given for work index variations which arise when laboratory grindability and impact crushing tests give different work index values.

by FRED C. BOND

Metal Wear

METAL wear is usually the second largest single item of expense in conventional grinding, and in wet-grinding installations it may approach or even exceed the power cost.

Metal wear is commonly expressed in lb/ton crushed or ground. However, variations in feed and product sizes and in work index are eliminated by expressing metal consumption as lb/kWh, including rejected worn parts; it can be obtained from the pounds of metal consumed per ton crushed or ground and the kWh/ton.

The pounds of metal both worn away and scrapped as worn parts varies with the abrasiveness of the ore and the abrasion resistance of the metal. The average values from a large number of mills with ordinary mill linings and grinding media, grinding wet a large range of siliceous ores, are as follows:

Mill Type	Rod	Ball
Media: lb/kWh	0.21 rods	0.14 balls
Lining: lb/kWh	0.026	0.020

The metal wear in dry grinding averages about one-seventh that of wet grinding the same material. However, many of the materials ground dry are softer and less abrasive than those ground wet. Ball coating in dry grinding can reduce the metal wear still further.

Well over half of the metal wear in wet grinding results from corrosion, or dissolution from the active nascent metal surfaces continually being produced in the mill. For this

reason the metal savings resulting from the use of hard alloy mill linings or media are usually greater in dry grinding than in wet grinding.

Abrasion Tests

Abrasion tests to indicate metal wear are made as follows:¹⁰ 400 grams of $-\frac{3}{4} + \frac{1}{2}$ -in. particles of the material to be tested are placed in a rotating drum for 15 min. The drum showers the rock through a rapidly rotating impeller contained within it. The impeller rotates at 632 rpm and consists of a $3 \times 1 \times \frac{1}{4}$ in. steel paddle with 2 sq. in. exposed to wear, machined of SAE 4325 steel hardened to 500 Brinell. The grams of weight lost by the paddle after impacting four successive 400-gram batches of rock for 15 min. each is called the abrasion index and designated as *Ai*. The combined product of the four 15-min. periods is screen analysed; it averages 80 per cent passing 13,250 microns.

Paddles of special alloy steels and cast irons can be tested with a standard abrasive material to determine relative wear resistance of the metals.

The average *Ai* values of some typical materials from 125 tests are listed in Table IIA in the appendix. A firm correlation with actual wear rates in crushers and grinding mills has not been made as yet; however, it is apparent that the abrasion index variations shown in Table IIA are much greater than the variations to be expected in the metal wear rates in commercial machines. It is observed that any correlation between the work index and the abrasion index is very slight.

Preliminary indications are that in wet grinding the pounds of balls per kWh equals (*Ai* + 1) divided by a

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number which ranges from 6 to 9, and liner wear is about one-eighth of ball wear. For wet rod mills the divisor ranges from 4 to 6, and liner wear is about one-seventh of rod wear. Wear in dry grinding is about one-seventh of wet grinding. The average metal wear in crushing is roughly comparable to the liner wear in wet grinding.

Volume of Grinding Charge

The fraction V_p of the total interior mill volume occupied by the grinding charge can be found by Equation (15), where D is the inside diameter and Q is the vertical distance down from the inside top of the mill to the levelled grinding charge: Q should be the average of measurements at the centre and both ends of the mill.

$$V_p = 1.13 - 1.26 Q/D \quad \dots (15)$$

The weight of the grinding charge can be calculated from D , V_p , and the inside length of the mill.

Loose round balls without breakage weigh 290 lb/cu. ft, loose rods 390 lb and silica pebbles 100 lb. Measurements of the weight of steel balls contained in a cubic foot box are usually less than 290 lb because of the increased void space at the sides of the box, and the weight contained decreases as the ball size increases. For accurate measurements the smallest dimension of the container should be at least 20 times that of the largest ball. Broken balls can reduce the specific weight to 280 lb/cu. ft, and broken rods to 340 lb. The specific weight of both ball and rod charges can also be reduced by a content of hard particles of the material being ground.

Tons of new balls T_b contained in a mill are found approximately from Equation (16), and tons of new rods T_r from Equation (17).

$$T_b = V_p D^2 L / 8.4 \quad \dots (16)$$

$$T_r = V_p D^2 L / 6.8 \quad \dots (17)$$

A charge of grinding balls contains approximately 40 per cent of void space, and rods in linear contact contain 20 per cent voids. Broken ore contains approximately 40 per cent void space, and weighs 100 lb/cu. ft multiplied by its specific gravity over 2.65. At 80 per cent solids or more the voids in a grinding charge of steel balls can contain 14 per cent of the ball weight in ore multiplied by $S_g/2.65$. A rod charge with the rods in linear contact can contain 7 per cent of the rod weight. However, both balls and rods are commonly forced apart in the mill and may contain more ore than these minimum amounts.

The per cent solids contained in the pulp within a wet grinding mill is ordinarily more than that entering and leaving the mill, since the water flows through the mill faster than the heavier particles of ore. This difference is increased in grate and peripheral discharge mills.

Critical Mill Speed

The theoretical critical speed N_c in rpm is the speed at which a particle of no diameter against the mill lining with no slippage would centrifuge. It is found from Equation (18), where D is the interior mill diameter in feet.

$$N_c = 76.6 / \sqrt{D} \quad \dots (18)$$

The fraction C_s of the critical speed represented by any mill rpm is found from Equation (19).

$$C_s = 0.01305 \times \text{rpm} \times \sqrt{D} \quad \dots (19)$$

Tumbling mills are usually designed to revolve at a constant fraction of the critical speed. Increased speed increases the mill capacity and power draught, but also increases the metal wear and maintenance cost per ton ground. For conventional wet grinding ball mills more than 8 ft in diameter, with peripheral lifters and a ball charge of more than 30 per cent of the interior mill volume, the maximum practical mill

speed is about 77 per cent of critical; for wet rod mills it is about 70 per cent of critical. Somewhat slower speeds are often found to be more economical.

Dry-grinding mills and pebble mills usually operate at about the same speeds as wet mills, with the maximum less limited than in wet mills.

Laboratory studies have shown that on the rising side of the mill each circular row of grinding balls slips downward with respect to the next outer row it rests upon, thus causing some grinding in this portion of the mill. A similar slippage is not observed in rod mills.

At speeds faster than 60 per cent of critical the smaller balls or rods in a grinding charge tend to concentrate with the pulp near the lining of a cylindrical mill, and the larger media are displaced toward the centre of the charge. Advantage is taken of this in ball mills to move the smaller balls toward the discharge end of the mill by spiral lifters trailing toward the mill discharge; or by making the mill shell slightly conical, with the smallest diameter at the discharge end.

Small-diameter mills are commonly operated at somewhat higher fractions of their critical speeds than are large mills, indicating that the proper mill speed is intermediate between a constant fraction of critical speed and a constant peripheral speed. An approximate empirical equation for the maximum practical rpm of wet ball mills, designated as N_o , is:

$$N_o = 57 - 40 \log D \quad \dots (20)$$

Effect of Mill Diameters

At a constant volume fraction V_p the mass of the grinding charge varies as D^2 . At a constant fraction C_s of the critical speed the peripheral speed varies as \sqrt{D} . It follows that the power input to a conventional tumbling mill theoretically varies as the interior diameter to the 2.5 power.

Measurements have shown that the wet fine grinding mill capacity varies as $D^{2.6}$. The diameter exponent increases slightly in mills operated under high impact conditions. Theoretically, the maximum exponent under reduction entirely by impact is 3.0.

Measurements have also shown that the power input exponent actually varies as $D^{2.4}$. The decrease from the theoretical 2.5 exponent probably results from energy from the falling balls or rods being transferred back to the mill shell on its down-going side. The actual diameter exponent per ton of grinding media is 0.4 instead of the theoretical 0.5.

The difference between the two observed diameter exponents of 2.6 and 2.4 is 0.2, which is the exponent defining the mechanical advantage of large-diameter mills. Mechanical efficiency increases as the interior mill diameter to the 0.2 power, and the kWh/ton required to grind decreases in the same ratio. Since the standard work index is based on mills of 8 ft interior diameter, the computed kWh/ton (W) by Equation (1) for any mill of D ft interior diameter should be multiplied by $(8/D)^{0.2}$.

Power per Ton of Grinding Media

The power input required in tumbling mills is calculated from the power required per ton of grinding media under the mill operating conditions. It varies with the fraction V_p of the mill volume occupied by the grinding charge, the fraction C_s of the critical speed, and the interior mill diameter D in feet.

Equation (21) gives K_{wr} , the mill input kW per ton of new grinding rods in conventional wet grinding overflow rod mills.

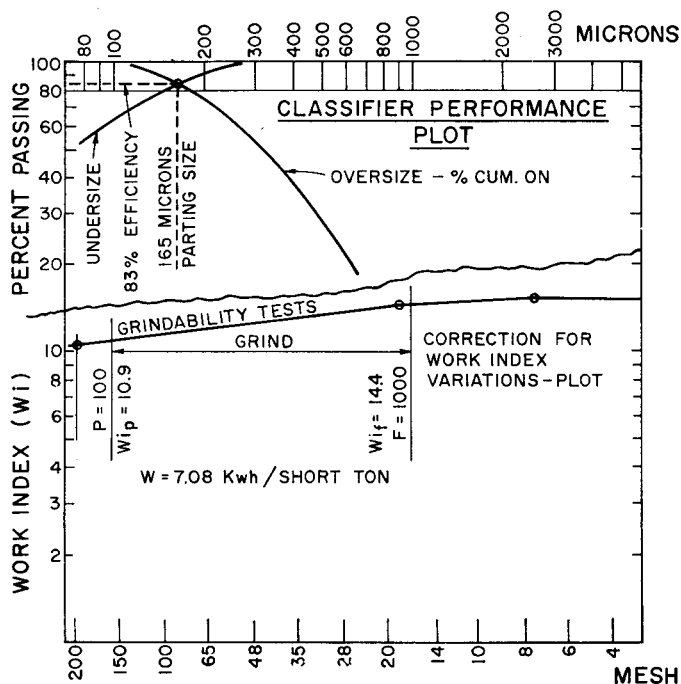
$$K_{wr} = D^{0.4} (6.3 - 5.4 V_p) C_s \quad \dots (21)$$

An accumulation of broken rods in the mill can reduce the actual power drawn by as much as 10 per cent.

Fig. 2.

TOP: Classifier performance plot. Oversize per cent cum. on plotted against undersize per cent passing. Intersection shows 83 per cent classifier efficiency and 165 microns parting size.

BOTTOM: Correction for work index variations. Grindability Tests— $W_i = 14.4$ at 14 mesh. $W_i = 10.4$ at 200 mesh. Grind in ball mill from 80 per cent passing 1000 microns to 80 per cent passing 100 microns. $W_{ip} = 10.9$, $W_{ir} = 14.4$; from Equation (42) $W = 7.08$ kWh per short ton.



Equation (22) gives Kwb , the mill input kW per ton of grinding balls for conventional wet-grinding ball mills using make-up balls larger than about one-eightieth of the mill diameter.

$$Kwb = 2.8 D^{0.4} (3.2 - 3 Vpd) Cs (1 - 0.1/2^{9-10Cs}) \dots (22)$$

For dry-grinding grate discharge mills Kwb should be multiplied by 1.08.

If Qd represents the vertical distance in feet from the inside top of the mill to the lowest discharge point, then

$$Vpd = 1.13 - 1.26 Qd/D \dots (23)$$

where Vpd is the fraction of the interior mill volume below discharge level. For a full low-level grate discharge Vpd equals 0.029. For wet-grinding grate and low-level discharge

mills, multiply Kwb by $\left[1 + \frac{0.40 - Vpd}{2.5}\right]$.

Slump Correction

Large-diameter ball mills fed with small make-up balls lose power because of excessive downward slippage of the ball charge on the rising side of the mill, and this loss of power input decreases the mill capacity. An empirical slump subtraction quantity S_s is computed for wet overflow and grate ball mills by Equation (24), to be subtracted from the Kwb value of Equation (22). No subtraction is made for wet ball mills with $D < 8$ or for dry ball mills with $D < 10$. For dry mills with $D > 10$ the slump quantity to be subtracted from Kwb is three-quarters of S_s as found by Equation (25), where B is the diameter of the make-up balls fed in inches.

$$S_s = \left(\frac{12D}{10B} - 8\right)^{\frac{1}{2}} \dots (24)$$

Effect of Reduction Ratios

The reduction ratio Rr is the ratio of the size of the new mill feed to that of the final product, or F/P . With scalped feed, F_c/P should be used. Rod mills are particularly sensitive to unfavourable reduction ratios, and if F/P is smaller than about 12 or larger than about 20 the kWh/ton (W) required for grinding increases. If Rro represents the optimum rod mill reduction ratio, its approximate value is found from

$$Rro = 8 + 5L/D \dots (25)$$

When the actual reduction ratio Rr is much smaller or larger than Rro , the work input W from Equation (1) should be multiplied by the empirical adjustment factor A_2 , where

$$A_2 = 1 + 2(Rr - Rro)^2/300 \dots (26)$$

Since peripheral discharge rod mills have low reduction ratios, they normally require an increase in W .

Ball mills are less sensitive to changes in reduction ratio than rod mills. However, when Rr becomes less than about 3, particularly in the fine grinding of concentrates, W from Equation (1) should be multiplied by the empirical adjustment factor A_3 , where

$$A_3 = \frac{20(Rr - 1.35) + 2.60}{20(Rr - 1.35)} \dots (27)$$

Free Vertical Oscillation

A body in free vertical oscillation falls from its highest position under the influence of gravity and is stopped forcibly by an equal deceleration; it is then accelerated upward at the same rate and reaches the same highest position as before. It describes a simple harmonic motion over a vertical distance of h inches with cpm cycles per minute, where

$$\sqrt{h} = 295/cpm \dots (28)$$

The critical frequency of vibrating bodies is calculated from Equation (28); as the cpm (or rpm) decreases below the critical, the tendency to maintain free vertical oscillation causes the amplitude h to increase.

The power necessary to maintain free vertical oscillation is directly proportional to the period of oscillation ($1/cpm$) and to \sqrt{h} . Where Kvo is the kilowatts necessary to maintain a ton mass in free vertical oscillation, then

$$Kvo = 328/cpm = 1.11 \sqrt{h} \dots (29)$$

The vertical component of the motion of a particle of no diameter against the lining of a ball mill at critical speed corresponds to free vertical oscillation with $h = 12D$, and $cpm = rpm$.

Effect of Oversize Feed

Feed particles which are too large for the grinding balls or rods to break are gradually worn down in the mill with a considerable loss of grinding efficiency. If the mill product

is coarse, the loss in mill capacity can be quite large; it decreases as the proportion of the total work consumed in fine grinding increases. The decrease in capacity caused by oversize grinding media is not as pronounced as that caused by oversize feed.

When F_o represents the maximum 80 per cent passing feed size in microns which does not appreciably decrease the grinding efficiency (is not oversize) with the ball or rod size calculated from Equation (12) or (13), and the work index is 13, F_o is about 4000 for ordinary ball mills and 30,000 for rod mills.

$$\text{For ball mills } F_o = 4000 \sqrt{(13)/Wi} \quad \dots (30)$$

$$\text{For rod mills } F_o = 30,000 \sqrt{(13)/Wi} \quad \dots (31)$$

The work input W for oversize feed ($F > F_o$) is calculated from

$$W = \left[\frac{10 Wi}{\sqrt{P}} - \frac{10 Wi}{\sqrt{F}} \right] \left[\frac{Rr + (Wi - 7)(F - F_o)/F_o}{Rr} \right] \quad \dots (32)$$

Grind Differential

The grind differential Gd evaluates the difference in the particle sizes of the concentrate and tailing when grinding for mineral unlocking and concentration by flotation or by gravity. An increased grind differential can be of major importance in such grinding circuits.

Where P is the 80 per cent passing size in microns of the feed to concentration, C_p is the 80 per cent passing size of the concentrate, and C_w is the per cent weight of the concentrate divided by the feed, the grind differential¹¹ is

$$Gd = \frac{P(150 - C_w) - 50 C_p}{P(50 - C_w) + 50 C_p} \quad \dots (33)$$

A typical copper flotation plant has a grind differential of 1.34. Any change in the grinding circuit which would increase the grind differential would result in relatively coarser grinding of the gangue, and should favourably affect costs, recovery and grade.¹¹

When making two grinds to 80 per cent passing a given product size P , with the first grind differential Gd_1 larger than the second grind differential Gd_2 , the relative mechanical efficiency of the first grind to the second is:

$$\text{Rel. Eff.} = \sqrt{1 + \left(\frac{Gd_1 - 1}{Gd_1 + 1} \right) - \left(\frac{Gd_2 - 1}{Gd_2 + 1} \right)} \quad \dots (34)$$

If the recession factor Rf is the number of standard $\sqrt{2}$ screen scale spaces between size P and size C_p , then

$$Rf = (\log P - \log C_p) / \log \sqrt{2} \quad \dots (35)$$

$$P = C_p (2)^{\frac{Rf}{2}} \quad \dots (36)$$

The grind differential Gd can be calculated from the recession factor Rf and the per cent weight of the concentrate C_w by substituting the value of P from Equation (36) into Equation (33).

The Schuhmann slope α of the heavier concentrate is commonly greater than that of the lighter tailings.

Classifier Performance

In closed-circuit reduction the efficiency and the separating size, cut point or parting size of the classifier or screen are important. The efficiency is commonly expressed as 100 minus the per cent of finished material or "unders" in the oversize returned to the mill or crusher, at a certain selected sieve size.

The parting size is defined as the size at which the per cent "unders" in the separator oversize equals the per cent "overs" in the separator undersize. The separator undersize per cent passing, and the separator oversize per cent cumulative retained on, are plotted as smooth curves on

the same linear or logarithmic graph sheet. The point where the two lines cross is the parting size, and the per cent passing at the crossing point is the per cent efficiency of the separator.

Percentage Circulating Load

In closed-circuit reduction the per cent circulating load (100 Cl) is 100 times the ratio of the weight of separator oversize returning to the reduction machine to the weight of the new feed entering the circuit in the same time interval.

P = microns 80 per cent of separator undersize passes.

F_p = per cent of new feed passing size P .

Dp = per cent of machine discharge passing size P .

Rp = per cent of separator oversize passing size P .

When the new feed enters the reduction machine

$$Cl = (80 - Dp)/(Dp - Rp) \quad \dots (37)$$

When the new feed enters the separator

$$Cl = (80 - F_p)/(Dp - R_p) \quad \dots (38)$$

The quantities required in Equations (37) and (38) can be determined from log-log or exponential plots of complete size distribution analyses. However, the screen analyses available may not permit determination of these quantities, and Cl must be calculated from the old equations¹⁴ based on the percentages passing 200 mesh or any other available fine screen.

If c represents the per cent of the classifier fine product passing 200 mesh, t is the per cent of the classifier coarse product passing the same size, and m is the per cent of the mill discharge passing the same size, then when the new feed enters the grinding mill

$$Cl = (c - m)/(m - t) \quad \dots (39)$$

When the new feed enters the classifier, and f_a represents the per cent of the new feed passing 200 mesh, then

$$Cl = (c - f_a)/(m - t) \quad \dots (40)$$

Any other suitable screen size may be substituted. However, the Cl values calculated from different screen sizes usually show a wide variation, and the mesh size at which the circulating load is calculated should be specified.

Per Cent new Classifier Feed to Closed-circuit Mill

When a rod mill in open circuit discharges to a classifier in closed circuit with a ball mill, the per cent of the rod mill discharge which enters the ball mill as scalped new feed equals

$$100 - \left[\frac{\% \text{ of rod mill discharge}}{\% \text{ passing parting size}} \right] \left[\frac{\% \text{ Eff.}}{100} + \frac{100 - \% \text{ Eff.}}{100 Cl} \right] \quad \dots (41)$$

Closed-circuit Versus Open-circuit Ball Mill Grinding

The listed work index values apply to ball mills grinding wet in closed circuit. For dry grinding in closed circuit, the work input W should be multiplied by 1.30.

The conversion to open-circuit grinding, either wet or dry, is done by multiplying the closed-circuit work input W by an open-circuit multiplication factor. In this connection the circulating load Cl should be unity or greater.

This factor varies with the reference per cent passing, or the per cent passing at which the open- and closed-circuit grinding are compared. For instance, if the reference per cent passing is 95, the multiplication factor will be much larger than at 80 per cent passing. If it is required that 95 per cent of the product passes 200 mesh the change from closed circuit to open circuit will require a much greater increase in power than if it is required that 80 per cent pass 200 mesh, or any other specified mesh size.

Some approximate open-circuit multiplication factors for various reference percentages passing are listed below in Table IV. They follow an exponential function.

TABLE IV

Reference per cent passing	Open-circuit multiplication factor
50	1.035
60	1.05
70	1.10
80	1.20
90	1.40
92	1.46
95	1.57
98	1.70

Correction for Work Index Variations

When an ore has natural grain sizes, it sometimes happens that laboratory rod mill and ball mill grindability tests and impact crushing tests give different work index (*Wi*) values

on the same sample at different product sizes. This complicates the calculation of the work input required (*W*) in kWh/ton from Equation (1), which is used with the desired capacity to find the required motor and mill size.

When this happens the various work index values are plotted vertically as ordinates on log-log paper against the 80 per cent passing sizes as abscissae, and the points are connected by straight lines which are extended horizontally to the edges of the paper. The 80 per cent passing sizes of grindability test products are found by dividing the screen openings *P*₁ in microns by log 20 (1.301), or by the values given following Equation (8) for fine ball mill tests. The 80 per cent passing size is 1.5 in. for the impact crushing tests.

The work index values at the designated 80 per cent passing product size *P* and feed size *F* are found from the plot and designated as *Wi_p* and *Wi_f*. Then the work input *W* is found from Equation (42) as given below:

$$W = \frac{10 Wi_p}{\sqrt{P}} - \frac{10 Wi_f}{\sqrt{F}} + \frac{10(Wi_f - Wi_p)}{1.5\sqrt{F}} \dots (42)$$

Appendix

TABLE 1A

Symbol*	In Equation No. †	Description	Symbol*	In Equation No. †	Description
<i>A</i>	4	Exponential plot slope.	<i>k</i> ₁₀₀	2, 3	Schuhmann microns 100 per cent passes.
<i>A</i> ₁	—	Fine product adjustment factor.	<i>L</i>	16, 17	Length of mill interior in feet.
<i>A</i> ₂	26	Rod mill <i>Rr</i> adjustment factor.	<i>Li</i>	3	Grind limit in microns.
<i>A</i> ₃	27	Ball mill <i>Rr</i> adjustment factor.	<i>m</i>	39, 40	Per cent passing in mill discharge.
<i>B</i>	—	Diam. in inches of balls and rods fed.	<i>Nc</i>	18	Critical mill speed in rpm.
<i>Bl</i>	5	Blaine surface area in sq. cm/gram.	<i>No</i>	20	Optimum mill speed in rpm.
<i>b</i>	4	Exponential size distribution coefficient.	<i>Oss</i>	10	Open side crusher setting in inches.
<i>C</i>	6	Ft.-lb per inch, impact crushing strength.	<i>P</i>	—	Product size—microns 80 per cent passes.
<i>Cl</i>	9, 37, 38, 39, 40, 41	Per cent circulating load/100.	<i>P</i> ₁	13, 14	Sieve opening in microns—A-C grindability.
<i>Cp</i>	33, 35, 36	Microns 80 per cent of concentrate passes.	<i>Q</i>	15	Feet from top of mill to charge.
<i>Cr</i>	—	Total crack length—cm/cc.	<i>Qd</i>	23	Feet from top of mill to lowest discharge point.
<i>Cs</i>	12, 13, 19	Fraction of mill critical speed.	<i>ρ</i>	3	Density.
<i>Css</i>	11	Close side crusher setting in inches.	<i>Rf</i>	35, 36	Recession factor between concentrate and feed.
<i>Cw</i>	33	Per cent weight of concentrate in feed.	<i>Rp</i>	37, 38	Per cent of oversize passing size <i>P</i> .
<i>c</i>	39, 40	Per cent passing in classifier fine product.	<i>Rr</i>	4, 5	Reduction ratio = <i>F/P</i> .
<i>cpm</i>	28, 29	Cycles per minute in free vertical oscillation.	<i>Rro</i>	25	Optimum rod mill reduction ratio.
<i>D</i>	19, 21-25	Inside mill diameter in feet.	<i>Sc</i>	6	Surface area in sq. cm/gram.
<i>Dp</i>	27, 38	Per cent of discharge passing size <i>P</i> .	<i>Sg</i>	24	Specific gravity of ore.
<i>d</i>	1	Microns 80 per cent passes.	<i>Ss</i>	5	Ball slump subtraction quantity.
<i>Ecc</i>	11	Eccentric throw of crusher in inches.	<i>Sw</i>	16	Sq cm per gram, Wagner surface.
<i>E</i>	4b	Exponential plot exposure ratio.	<i>Tb</i>	17	Tons of balls in mill.
<i>Er</i>	1, 2, 3, 4, 19, 32	Feed size—microns 80 per cent passes.	<i>Tr</i>	39, 40	Per cent passing in classifier coarse product.
<i>Fc</i>	—	Equivalent size of scaped feed.	<i>Vp</i>	15	Fraction mill volume occupied.
<i>Fo</i>	30, 31, 32	Maximum efficient feed size in microns.	<i>Vpd</i>	23	Fraction of mill volume below discharge level.
<i>Fp</i>	38	Per cent of new feed passing size <i>P</i> .	<i>W</i>	1, 2, 3	Work input in kWh per short ton.
<i>f</i>	5	Diameter of feed particle in microns.	<i>Wi</i>	1, 6-13	Work index: kWh/ton to 80 per cent—100 microns.
<i>fa</i>	40	Per cent passing in mill new feed.	<i>Wi</i> _f	1, 10	Work total: kWh/ton to product size.
<i>Gd</i>	33, 34	Grind differential.	<i>w</i>	4	kWh/ <i>Wi</i>
<i>Gbp</i>	8	Ball mill grindability A-C, net grams/revolution.	<i>X</i>	4	Exponential plot abscissa.
<i>Grp</i>	7	Rod mill grindability A-C, net grams/revolution.	<i>Y</i>	4	Per cent cumulative retained on any size.
<i>Hd</i>	9	Hardgrove grindability.	<i>z</i>	2, 3	Slope of size distribution line.
<i>h</i>	28	Inches of free vertical oscillation.	<i>Wi_p</i>	42	Work Index from plot at product size <i>P</i> .
<i>K</i>	—	Grinding media size constant.	<i>Wi_f</i>	42	Work Index from plot at feed size <i>F</i> .
<i>Kvo</i>	29	Kilowatts per ton in free vertical oscillation.			
<i>Kwb</i>	22	Kilowatts per ton of balls.			
<i>Kwr</i>	21	Kilowatts per ton of rods.			

*In this paper three classes of symbol are used: lower case single letters, upper case single letters and compound symbols consisting of an upper case letter followed by one or more lower case letters. They are distinguished by the sequence

rule; lower case single letters are written first, followed by upper case single letters, and in every term the compound symbols are written last. † Equations (2), (18), (39) and (40) were not derived by the author.

TABLE IIA—Abrasion Index Test Results

Abrasion index *Ai* is the fraction of a gram weight lost by the standard steel paddle in 1 hr of beating 1600 grams of ½ in. × ½ in. particles. The product averages 80 per cent passing 13,250 microns

Material	No. Tested	<i>P</i> Microns	<i>Wi</i>	Average <i>Ai</i>	Material	No. Tested	<i>P</i> Microns	<i>Wi</i>	Average <i>Ai</i>
Alnico	1			0.3850	Limestone	19	13,000	12.1	0.0256
Alumina	6	15,500		0.6447	Magnesite	3	14,400	16.8	0.0750
Asbestos cement pipe	1	13,330		0.0073	Magnetite	2		10.2	0.2517
Cement clinker	2	12,100	10.9	0.0409	Manganese ore	1		17.2	0.1133
Cement raw material	4		10.5	0.0372	Nickel ore	2		11.9	0.0215
Chrome ore	1	10,200	9.6	0.1200	Perlite	2			0.0452
Coke	1		20.7	0.3095	Pumice	7		11.9	0.1187
Copper ore	12	12,900	11.2	0.0950	Quartz	1		12.8	0.1831
Coral rock	1			0.0061	Quartzite	3		12.2	0.6905
Diorite	1		19.4	0.2303	Rare earths	1			0.0288
Dolomite	5		11.3	0.0160	Rhyolite	2	13,200		0.4993
Gneiss	1		20.1	0.5360	Schist-biotite	1		23.5	0.1116
Gold ore	2		14.8	0.2000	Shale	2	11,200	11.2	0.0060
Granite	11	15,200	14.4	0.3937	Slate	1		15.8	0.0179
Gravel	2		19.0	0.3051	Slag	1		13.8	0.1423
Hematite	3		8.6	0.0952	Slate	1		11.5	0.0001
Iron ore (misc.)	4		5.4	0.0770	Sulphur	7		16.2	0.6837
Lead zinc ore	3		8.3	0.1520	Taconite	11	14,900	19.9	0.3860
					Trap Rock				
					(Total)	125(Av.)	13,250	13.8	0.228

TABLE IIIA—Average Work Indexes by Types of Materials

Caution should be used in applying the average work index values listed here to specific installations, since the individual variations between materials in any classification may be quite large.

Material	No. of Tests	Average		Material	No. of Tests	Average	
		Specific Gravity <i>S_g</i>	Work Index <i>W_i</i>			Specific Gravity <i>S_g</i>	Work Index <i>W_i</i>
All materials tested	2088	—	13.81	Kyanite	4	3.23	18.87
Andesite	6	2.84	22.13	Lead ore	22	3.44	11.40
Barite	11	4.28	6.24	Lead-zinc ore	27	3.37	11.35
Basalt	10	2.89	20.41	Limestone	119	2.69	11.61
Bauxite	11	2.38	9.45	Limestone for cement	62	2.68	10.18
Cement clinker	60	3.09	13.49	Manganese ore	15	3.74	12.46
Cement raw material	87	2.67	10.57	Magnesite, dead burned	1	5.22	16.80
Chrome ore	4	4.06	9.60	Mica	2	2.89	134.50
Clay	9	2.23	7.10	Molybdenum	6	2.70	12.97
Clay, calcined	7	2.32	1.43	Nickel ore	11	3.32	11.88
Coal	10	1.63	11.37	Oil shale	9	1.76	18.10
Coke	12	1.51	20.70	Phosphate fertilizer	3	2.65	13.03
Coke, fluid petroleum	2	1.63	38.60	Phosphate rock	27	2.66	10.13
Coke, petroleum	2	1.78	73.80	Potash ore	8	2.37	8.88
Copper ore	308	3.02	13.13	Potash salt	3	2.18	8.23
Coral	5	2.70	10.16	Pumice	4	1.96	11.93
Diorite	6	2.78	19.40	Pyrite ore	4	3.48	8.90
Dolomite	18	2.82	11.31	Pyrrhotite ore	3	4.04	9.57
Emery	4	3.48	58.18	Quartzite	16	2.71	12.18
Feldspar	8	2.59	11.67	Quartz	17	2.64	12.77
Ferro-chrome	18	6.75	8.87	Rutile ore	5	2.84	12.12
Ferro-manganese	10	5.91	7.77	Sandstone	8	2.68	11.53
Ferro-silicon	15	4.91	12.83	Shale	13	2.58	16.40
Flint	5	2.65	26.16	Silica	7	2.71	13.53
Fluorspar	8	2.98	9.76	Silica sand	17	2.65	16.46
Gabbro	4	2.83	18.45	Silicon carbide	7	2.73	26.17
Galena	7	5.39	10.19	Silver ore	6	2.72	17.30
Garnet	3	3.30	12.37	Sinter	9	3.00	8.77
Glass	5	2.58	3.08	Slag	12	2.93	15.76
Gneiss	3	2.71	20.13	Slag, iron blast furnace	6	2.39	12.16
Gold ore	209	2.86	14.83	Slate	5	2.48	13.83
Granite	74	2.68	14.39	Sodium silicate	3	2.10	13.00
Graphite	6	1.75	45.03	Spodumene ore	7	2.75	13.70
Gravel	42	2.70	25.17	Syenite	3	2.73	14.90
Gypsum rock	5	2.69	8.16	Tile	3	2.59	15.53
Ilmenite	7	4.27	13.11	Tin ore	9	3.94	10.81
Iron ore	8	3.96	15.44	Titanium ore	16	4.23	11.88
Hematite	79	3.76	12.68	Trap rock	49	2.86	21.10
Hematite—specular	74	3.29	15.40	Uranium ore	20	2.70	17.93
Oolitic	6	3.32	11.33	Zinc ore	10	3.68	12.42
Limonite	2	2.53	8.45				
Magnetite	83	3.88	10.21				
Taconite	66	3.52	14.87				

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Research in Pelleting

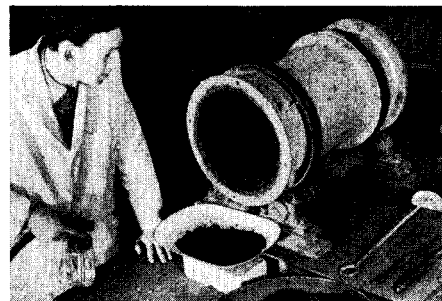
THE Research and Development Division of Head Wrightson has built two pelletizing drums for experimental work. Their research in this field is done in collaboration with Head Wrightson Stockton Ltd., who manufacture and market this equipment. A large drum and a small one have been constructed, and details are given below.

The slope and rpm of the larger drum are both variable, and as it is in fact of the same-size order as many "industrial" drums, it can be expected to be a most useful piece of equipment. It is roller mounted and if necessary and more convenient, could be taken to a site to study a client's problem on the spot.

The smaller drum is for bench work used to appraise the relevant properties of different powders and binders; and also to produce small batches of pellets for subsequent experimental work.

In the photograph, for example, Norwegian magnetite concentrates are being pelletized; enquiries have been received on the treatment of fume from gas cleaners

Small pelletizing drum installed at the laboratories of Head Wrightson's Research and Development Division.



following oxygen steelmaking furnaces, and work has started on this problem.

In general these drums are used to assess the possibility of making adequate pellets from any material and test work is directed to determining the appropriate: (a) degree of wetness; (b) pelletizing aid—or binder; (c) retention time.