EULER DECONVOLUTION, PAST, PRESENT AND FUTURE: A REVIEW

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INTRODUCTION

Euler's homogeneity relation has attracted sporadic interest from geophysicists over the years. It may be stated succinctly in the form

$$(x-x_0)\frac{\partial T}{\partial x}+(y-y_0)\frac{\partial T}{\partial y}+(z-z_0)\frac{\partial T}{\partial z} = N(B-T) \qquad [1]$$

where (x_o, y_o, z_o) is the position of a source whose total field T is detected at (x, y, z). The total field has a regional or background value B. N is the degree of homogeneity, interpreted physically as the fall-off rate with distance and geophysically as a structural index (*SI*: Thompson, 1982).

Profile analysis

Hood (1963) showed that Euler's relation could be used to calculate depth to point pole (*SI*=2) or point dipole (*SI*=3), given a measured vertical gradient. Ruddock *et al.* (1966) were awarded a U.S. patent describing the use of a vertical gradiometer and Euler's relation to determine the depth of and fall-off rate (*SI*) from a magnetic discontinuity. They recognized *SI*s of 1, 2 and 3 as corresponding to sheets, line sources and point sources, respectively.

The relation was subsequently employed to estimate source type, given position and depth known or estimated by other methods (Slack *et al.*, 1967; Barongo, 1984). Steenland (1968) has pointed out that fall-off rate (*SI*) is only approximately a constant for real source bodies over particular distance ranges.

Thompson (1982) developed the profile technique quite fully, named it EULDPH, and suggested that SIs between 0.5 and 3 were useful on pole reduced magnetic data. The fault model (*SI* 0.5) required some empirically based corrections to obtain depth. Soon thereafter it was applied to data from the Witwatersrand Basin (Durrheim, 1983; Wilsher, 1987; Corner and Wilsher, 1989). Wilsher also showed, by application of Poisson's relation, that the vertical gradient of gravity (i.e., pseudo-magnetic field) could be expected to behave like magnetic field and could benefit from EULDPH methods.

Grid analysis

Reid *et al.* (1990) followed up a suggestion in Thompson's paper and developed the equivalent method operating on gridded magnetic data. They also introduced the concept of the zero *SI* for contacts. Finally, they suggested that the technique could be expected to work on gravity data by showing that Euler's equation was approximately obeyed by the gravity anomaly over a finite step using an *SI* of 1.0. They coined the term "Euler deconvolution" by analogy with "Werner deconvolution".

Paterson *et al.* (1991) showed that an *SI* value of 2.0 was of practical use in locating kimberlite pipes. Since then, SIs between 2 and 3 have found environmental application in drum location (Yaghoobian *et al.*, 1992).

Regional studies

Euler deconvolution has found wide application to regional studies. These include the Witwatersrand Basin (Durrheim, 1983; Corner and Wilsher, 1989), the Ashanti Gold Belt of Ghana (Beasley and Golden, 1993), Wales (McDonald *et al.*, 1992), and the Sudbury structure (Hearst and Morris, 1993).

Gravity deconvolution

Because vertical gradient of gravity is effectively a pole reduced pseudo-magnetic field, Euler deconvolution should be directly applicable to vertical gravity gradient (Wilsher, 1987). Marson and Klingele (1993) have shown excellent examples of this on both model and small-scale archeological data.

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RECENT DEVELOPMENTS

Defocused solutions

The clean-up of sprays of defocused solutions, to which the method seems prone, has been addressed with some success by Fairhead *et al.* (1994) who applied a Laplacian filter to obtain those portions of the grid that showed significant curvature and restricted the deconvolution to those areas, so eliminating most spurious solutions. Kuttikul (1995) has shown that the sprays contain information about interface dip (to which Euler deconvolution is otherwise insensitive). He also showed that positive curvature of the Analytic Signal is a useful discriminator against such sprays. Featherstone (Featherstone, P.S., pers. comm., 1995) has suggested that the sprays may be regarded as diffractions and may possibly be collapsed onto their prime points using seismic migration techniques. Huang (Huang, D., pers. comm., 1996) has shown that it is possible to improve solution clustering by exploiting the clustering of gradient vector intersections.

Solution for *SI* and multiple sources

The choice of structural index remains a vexing problem, because structures are poorly imaged and depths are biased if the wrong index is used for any given feature (Reid *et al.*, 1990). In any real geological situation, features representing more than one structural index are likely to be present. Neil (1990) and Neil *et al.* (1991) had some success in deriving structural indices as well as positions from the data themselves, using statistical methods that normalised the uncertainties on each position and depth solution. Stavrev (1997) has demonstrated the use of Similar Transformations to reduce ambiguity in choosing *SI* and in locating multiple sources.

It may be possible to avoid the issue by using a multiple source approach. Hansen (Hansen, R.O., pers. comm., 1995) recently produced a theoretical treatment of this fairly intractable problem. Although it may not be simple to implement, it offers the possibility of viewing all bodies as composites of the contact ($SI = 0$) case. If this approach is successful, it could address the additional problems of fractional and variable *SI* posed by many real cases (Steenland, 1968; Ravat, 1994). A multiple source approach also offers the possibility of dealing with higher order backgrounds.

Fractal dimension

Ravat (1994) has pointed out that the definitions of *SI* and Fractal Dimension are effectively identical. He has exploited this to examine problems of variable *SI*, but the insight may lead to other developments.

Other fields

Huang (1996) shows that Euler deconvolution can be applied to any field or function displaying Euler Homogeneity. This includes the Analytic Signal and Horizontal Gradient of gravity or magnetic fields when they are themselves homogeneous. The appropriate *SI* will be that applicable to the gravity or magnetic case, plus unity.

Visualisation

Useful work was done on visualising the results on a graphics workstation by Allsop *et al.* (1991). The 3-D feel of a rotating cloud of points was displayed in real time and captured on video. A graphics workstation approach using visualisation tools such as AVS offers obvious further benefits which are being investigated.

Abuse of Euler deconvolution

I have observed a great many examples of abuse of the method, arising from naive use of both commercial and home-written software. The main problems seem to be poor gradient grids and a lack of understanding of the meaning of *SI*. Poor gradient grids frequently occur when they are derived using Fourier methods without due care to avoid the ringing to which the methods are prone. The only necessary precaution is critical imaging of the gradient grids before use. Some workers appear to regard *SI* as a “Fudge Factor” completely at the choice of the experimenter, without any consideration of the implicit physical and geological meaning of any particular choice. Misleading results are the inevitable consequence.

THE FUTURE

1. Develop a reliable method of estimating structural index or avoiding the problem.
2. Discover the meaning and uses of the background value, B.
3. Develop a better understanding of the deconvolution of gravity.
4. Investigate multiple source deconvolution or other means of dealing with higher order backgrounds.
5. Develop visualisation and on-screen interpretation using modern graphics workstations.
6. Explore the use of Euler deconvolution on other homogeneous fields.

We are presently working on these topics and hope to have results to report before long.

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